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A critical review of the capital investment decision making process

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A CRITICAL REVIEW OF THE CAPITAL INVESTMENT DECISION MAKING PROCESS

by
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Investments:[1]

It is a serious and noble act. It is based on technology that is "like the earth--it does not lie."

It is a promise of the future; it requires courage and personal discipline.

It is a collective activity and a condition of growth; without growth the morale of managers cannot be high.

1 International Report on Factors in Investment Behavior, published by the Organization for Economic Cooperation and Development, Paris, France. Compiled by Bruce R. Williams, University of Manchester, The French Report, Page vi.12.

ABSTRACT

This thesis attempts to untangle the complexities of capital investment decision making by an in depth review and analysis of the literature on the subject which has grown since World War II in discontinuous and widely spread bits. Then, this author tries to weave these isolated threads of knowledge into some kind of comprehensive whole.

The major facets of the capital investment process which are so treated are: uncertainty and risk, the many criteria employed for evaluating investments, the cost of capital and its many ramifications, utility theory, and game theory. Several recommendations for future research projects are also given. A brief review of game theory is included as an appendix.

PREFACE

This thesis deals with capital investments and the inherent decision making problems and processes faced by management. The complexity of the capital investment process is overwhelming. The lack of a basic theory has been the biggest handicap in the practice of capital budgeting. To help supply this important need, earlier researchers such as Irving Fisher and Lord Keynes, and recent theorists such as Joel Dean, Vera and Friedrich Lutz, and Ezra Solomon,[1] have structured a theoretical framework within which the decision maker could operate on a sounder and happier basis. The literature on capital budgeting has grown accordingly, but in discontinuous and widely spread bits. To the best of our knowledge, no one has attempted so far to take these isolated threads of knowledge, unravel them and weave them into a comprehensive whole. It is the principal purpose of this thesis to undertake such an attempt. As a result, it is hoped that this thesis would be of assistance to the decision maker by helping to present a clearer understanding of the more important factors that must be considered in capital budgeting.

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[1] Refer to Appendix 4 for a more complete list of recent investigators.

CHAPTER I INTRODUCTION

1.1 General

Members of upper management are the strategists upon whom the survival[1] and success of the firm depends. They:

"1. Must decide what opportunities are to be pursued and what risks are acceptable.

2. Must decide on scope and structure, and especially on the right balance between specialization, diversification, and integration.

3. Must decide between time and money, between building its own or 'buying' -- i.e., using scale of a business, merger, acquisition and joint venture -- to attain their goal.

4. Must decide on an organization structure appropriate to its economic realities, its opportunities and its program for performance" (10, p. 203).

Investment decision making pervades directly or indirectly, each of these situations. Today, more than in the past, investment decisions, especially capital investments, are probably the most important and at the same time the most difficult decisions that confront top management.

Uncertainty, which is the main element in management investment decisions, exists because of the need to account for the future.

"We know only two things about the future:

a. It cannot be known.

b. It will be different from what exists now and from what we now expect.

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1 Industrial firms in general have a chance of only 24% of surviving five years and of only 19% of being in business ten years after their beginning. (21, p. 192)

> These assertions are not particularly new or particularly striking. But, they have far-reaching implications. Business these last ten or twenty years has accepted the need to work systematically in making the future. But long-range planning does not and cannot aim at the elimination of risks and uncertainties. That is not given to mortal man. The one thing he can try is to find, and occasionally to create, the right risk and to exploit uncertainty. The purpose of the work on making the future is not to decide what should be done tomorrow, but what should be done today to have a tomorrow" (10, p. 173).

Up to the present, most management decision making has been based on art, intuition, experience, and rules of thumb, perhaps because the scientific techniques, employed by operations research and other management sciences, have reached only as high as middle management and seldom, if ever, higher. The importance of upper management strategic decisions has caused several academic disciplines to focus on this problem. Some modi operandi which have been generated by science in its frontal attack on uncertainty are discounted cash flow, utility functions, game theory, linear and dynamic programming, and industrial dynamics.

1.2 Capital Investment -- Importance and Definition

The importance of capital investment can be best gauged by the sums of money invested each year in both profit making and non-profit making projects. It has been reported that in 1959:

> "$37 billion were spent on non-farm producers' plant and equipment. $5 billion were spent on farm equipment and on construction and $3 billion on other private construction including religious, educational, social and recreational, and hospital and instutional.
>
> $4 billion additional were invested in inventories. Expenditures for research and development totaled approximately $12 billion. Additional billions were invested in working capital by industry. Many additional billions were spent for capital purposes by

federal, state and local governments." (3, p. 185)

By 1965, investments of U. S. companies in plant and equipment alone were approaching $50 billion a year. Another $50 billion went into acquisitions, development of new products, and other investment expenditures;[1] all together, nearly 15% of the U. S. gross national product.

Capital investment is synonymous, from an operational point of view, with capital budgeting, which in turn can be defined as the planning of the use of dollars to purchase capital. Prof. Barish defines capital budgeting as expenditures "in which the company's funds are committed for projects which will return the invested funds and profits during future periods", (3, p. 185). "It should constantly be borne in mind that the budgeter sees the business from the point of view of the dollar of profit. For him, the business is not primarily producing goods. First of all, it produces dollars of sales at a cost per dollar resulting in a profit per dollar", (23, p. vi). Accordingly, capital is wealth and it is expected to produce more wealth.

Capital budgeting includes replacements, improvements and additions to plant and equipment. It also includes land for plant sites and intangibles such as patents, franchises, and trademarks. Research may also be considered a part of the capital budget, mainly because many research projects usually require several years for commercialization and are therefore capitalized expense.

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1 "Businessmen plan in 1966 to increase capital spending 15%", Time Magazine, December 31, 1965, "The Economy", p. 64.

There are at least two very essential characteristics of a capital budget or investment:

1. Uniqueness - "Each project constitutes a distinct income, gain, degree of urgency", (23, p. 233). Various techniques can be applied to appraise the value of a capital budget. However, each particular capital budget must also be considered for its unique characteristics to get a real understanding of its value.
2. Time - There are at least four different facets of time relative to budgeting.
   (a) One deals with the time period from conception of a project to its final write-off. Time is required for capital to be "weaned" to the point where it itself is productive. Then, from that point on, more time is required for the capital to produce an amount equal to the original investment and then profit.
   (b) Another time concept deals with long and short range budgeting. Generally, long range planning budgeting extends 3-5 years into the future and incorporates an estimate of the future amounts of funds that will be available, or should be available for capital expenditures in each year of a planning period. Accordingly, it is indicative of capital additions. It is by means of the planning budget that management can be certain

that the capital commitments of the firm are in agreement with its dynamic business environment.

The long range planning budget may or may not consist of individual proposals for capital expenditures. Usually, it covers areas of future expenditures rather than specific proposals for replacement or expansion of facilities. "A word of caution, however, is necessary. The long range planning budget must not become definitive. The capital expenditure proposals included in a five year forecast should remain only as concepts. It should not be required that a proposal be included in the long range budget before it can be included in the short range budget. It is difficult to believe that useful replacement expenditures can be forecasted that far in advance", (14, p. 27).

On the other hand, the short range finance or capital budget is the means by which management attempts to "harmonize" the cost of expenditure proposals with the available funds or needed funds, for the next year, or in some cases, the the next two years. The modus operandi of this harmonization depends upon the willingness and capability of the firm to supplement available capital funds by borrowing or by sale of additional equity securities. Accordingly, short range budgets, depending upon source, can be classified as follows:

(1) Rationing Budget - such a budget usually indicates the probable allocation of a given amount of expenditure. Its functions, internally generated, are limited to depreciation charges and retained earnings.

(2) Financing Budget - this type of budget uses external money sources, i.e., borrowing or issuing new equity securities. Generally, with this type of budget, all proposals which meet a predetermined minimum of economic worth will be implemented.

> "Obviously, a firm that has access to money markets and is willing to use them is better able to implement acceptable proposals than a firm entirely dependent on internally generated funds. However, firms that do not finance externally do not necessarily lack sufficient capital funds; a firm may generate enough capital funds from its depreciation charges and retained earnings to maintain its plant and equipment and provide for desired growth. This would seem more likely in a non-inflationary period, than in the period of rising prices. Continuing inflation causes the replacement cost of some facilities to be greater than the original historical cost. This means that the funds provided by a depreciation charge based on historical cost are usually insufficient for replacement. The situation is complicated by the fact that

> Internal Revenue Code requires the use of historical cost depreciation for federal tax purposes. The tax bill is higher when the depreciation deduction is based on historical costs than it would be if current costs were used, and the result is smaller after tax income for use in the business. Because the retained earnings after taxes plus the financial depreciation fail to provide the increased capital funds needed, replacement of facilities and expansion are hampered." (14, p. 30)

(c) The third facet of time can be viewed as a stochastic process. The problem posed by this concept to management deals with the acceptance or bypassing of a current investment opportunity in light of future possible investments available to the firm or the decision maker. For example, suppose that the firm is faced with the choice of investing now in an alternative which meets its acceptable critical investment opportunity index, e.g., present value or internal rate of return, or waiting until the next decision making date when an investment with twice the value of this marginal one will be available with a probability of .85. It may very well be to the firm's advantage to reject a present investment opportunity in hopes of using its disposable assets to accept a more profitable investment in the uncertain future. Of course, the amount of disposable assets, e.g., retained profits, and the amount of time such assets can remain unproductive are prime factors to be considered. The firm's decision makers, no doubt, subjectively take the above factors into consideration. But, how accurate, how analytical is their analysis? The analysis of such a problem involves dynamic programming. In a doctoral thesis, J. L. Fisher (82) summarizes his approach as follows:

> "As developed in this thesis, the decision-making problem consists of two distinct stages. Firstly, the decision maker is interested in determining optimal decision rules for accepting or rejecting investment opportunities. Secondly, given these optimal decision rules, the decision maker is interested in whether he should hold the disposable assets he currently has or should these disposable assets be transferred to some other activity where they will be more productive. The problem of determining the maximum amount of assets the decision maker should hold is examined in each of the cases studied. By simultaneously adjusting the discount factor and the probability of an offer, we can approach as closely as desired the continuous decision-making problem."

Another reference to dynamic programming for such a stochastic investment problem is (42).

(d) A fourth facet of timing involves possible revisions to planned budgets, usually the short-range budgets, to accommodate changing circumstances. The revision capabilities are usually inherent in the time period between budget planning sessions, whether it be quarterly or monthly or even weekly. Generally, the length of time between revisions of the short-range budget should be in inverse relationship to the degree of its definiteness. For example, when the budget includes proposals in conceptual form, and allowances are made for substitution of better proposals, the budget period need not be less than the fiscal year. On the other hand, in cases where only those proposals included in the budget will be implemented, the budget period should be as short as possible to permit faster implementation of proposals originated after the budget had been drawn up.

1.3 Who Are the Decision Makers[1]

Who decides which project is to be pursued and which project is to be bypassed? Who are the firm's decision makers?

In any corporation all authority ultimately rests with the board of directors; however, the degree of delegation of this authority varies from firm to firm. It is not unusual to find that in some firms, the board of directors has delegated this authority to act in capital expenditure completely; i.e., no capital expenditure proposals are approved by the board. Antithetical to this approach, we also find some boards of directors finally approving or rejecting all proposals.

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1 This section is largely based on the findings of Prof. Donald F. Istvan of Indiana University, who, in 1960, conducted an investment decision study of 48 of the country's largest firms. (14)

The reason usually given for this complete lack of delegation can be summarized as follows: Present capital expenditures are the framework from which future profits are created; the board of directors is directly responsible to the stockholders for these future profits; therefore, only the board of directors should be responsible for present capital expenditure. Such an attitude, however, contains a fallacy in reasoning. Since the board of directors is finally responsible to the stockholders for all actions undertaken by the firm, the premise should hold true not only for capital expenditures but for all decisions. It has been noted by the decision makers themselves that where approval for all proposals is retained by the board of directors, the administrative processing of the proposals may be slowed down by a bottleneck at that level.

Where there is a delegation of authority, it can rest in a committee composed of high level officers and individual members of the board, with an executive vice-president, with various divisional vice-presidents, or with certain functional managers. In some firms, the board of directors grants a blanket approval for a maximum amount of expenditure to a subordinate official of the firm. This official is then allowed to authorize expenditure of this amount of money. The plan allows a plant manager, for instance, to approve proposals that do not exceed $5,000 each, but he is limited, perhaps, to a $50,000 total of such approvals throughout the year. This technique can greatly facilitate the processing of many smaller proposals.

We usually find that the person or group to whom authority is delegated varies in relation to the amounts of money involved. A typical breakdown of authority and size of expenditure is given in the following table:

Size of Expenditure	Decision Maker
Over $200,000	Board of directors or specified top management committee
$100,000-$200,000	President or chairman of the board of directors
$25,000-$100,000	Vice-president in charge of division
$5,000-$25,000	Plant Manager
Under $5,000	Persons delegated by plant manager

It is interesting to note the results of Prof. Istvan's findings of 48 major firms of the United States which have been summarized as follows:

	Number	How Delegated
Authority Completely Delegated by Board of Directors	6	4-Committee 1-Executive Vice-President 1-Various Divisional Vice-Presidents
No Delegation of Authority	6	-
Various Degrees of Delegation	36	Usually in Accordance with Above Table

CHAPTER II THE ROLE OF UNCERTAINTY

Because "the only sure thing in this world is the past, but all we have to work with is the future"[1] in investment planning, a careful review of what is uncertainty and its role in capital investments is pertinent.

2.1 What is Uncertainty?

Although uncertainty and risk both refer to a situation in which future outcomes are imperfectly known, there is a definite difference between the two. The risk situation is characterized by the following:

a. The probabilities of all alternative possible outcomes are known.

b. It is repetitive in nature.

c. It possesses a frequency distribution from which observations can be drawn.

d. It is possible to generate inferences from these observations by objective, statistical methods.

As an example, almost any of the phenomena against which insurance can be drawn, can be classified as risks. The reason for this lies in the fact that "adequate" statistics have been compiled about the relative frequency of each event. Situations under uncertainty, on the other hand, possess the following criteria:

a. The situation is unique. There is no possibility of "experiment" replication.

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1 (18, p. 199).

b. Its frequency distribution cannot be objectively defined.

Most investment decisions belong to this classification. The general term uncertainty can be further differentiated into the two classes of "pure" uncertainty and "subjective" uncertainty. Under "pure" uncertainty or ignorance the decision maker is aware that one of several possible things is true but he does not know the relative probabilities of their truth or if it is even meaningful to talk about probabilities. This dilemma can be illustrated by an example employed by Savage: Suppose you are making an omelet, have broken five good eggs into a bowl, and contemplate a sixth, as yet unbroken egg. Two conditions or states are possible for this sixth egg: (State 1 = E1) = "good", (State 2 = E2) = "rotten". Now, you plan either to break the egg into the bowl with the five good eggs (Act 1 = A_1), or break it into a saucer for inspection (Act 2 = A_2), or throw it away without breaking (Act 3 = A_3). A final result or consequence matrix could be generated as follows:

	E_1 = "good"	E_2 = "rotten"
A_1	Six-egg omelet.	No omelet and five good eggs destroyed.
A_2	Six egg omelet and a saucer to wash.	Five egg omelet and a saucer to wash.
A_3	Five egg omelet and one good egg destroyed.	Five egg omelet.

Luce and Raiffa offer hypothetical conditions which could turn this case of "pure" uncertainty into one risk. "Suppose the husband - a scientifically minded farmer - 'knows' that in a random sample of six eggs, the conditional probability of the sixth egg's being rotten when the other five are good is 0.008. Thus, he may view breaking the sixth egg into the bowl as the lottery: 0.992 probability of the six egg omelet prize and 0.008 probability of the no-omelet and five good eggs destroyed prize. In other words, an a priori probability distribution over the states "good" and "rotten" allows one to structure the problem as one of decision making under risk - as a choice among lotteries", (16, p. 277). It is also interesting to note that in this case, the true state between E_1 and E_2 cannot be affected by any strategy. Either the egg is rotten or it is not rotten and the decision maker can do nothing about this.

Under a condition of "pure uncertainty, the strategy or action that a decision maker will employ will depend upon the criterion he uses. But, different criterion will usually generate different strategies. Let us investigate some of these criteria based upon attitude; e.g., pessimistic, optimistic, of the decision-maker.

Because a pay-off matrix will be germane to the overall analysis, let us consider the following payoff matrix:

	E_1	E_2	E_3	E_4
A_1	3	3	0	3
A_2	1	1	1	3
A_3	0	5	0	0
A_4	2	4	0	0

Let $E = \{E_j\}$ where E_j is a "state of the world". The set E is exhaustive and mutually exclusive. Savage defines the world "as the object about which the person is concerned", and a state of the world "as a description of the world, leaving no relevant aspect undescribed", (25, p. 9).

$A = \{A_i\}$ where A_i denotes a course of action; i.e., a function which assigns a payoff or consequence to each state of the world. All possible courses of action are represented by the matrix. As an example, if the state of the world is E_2 and the decision maker chooses action A_3, then his payoff is 5.

If the decision maker is an extreme pessimist he will assume that the worst possible consequence will result due to the most unfavorable state of the world. Accordingly, he would pick the smallest payoff in each row and then select the action in which this minimum is greatest. In other words, the criterion is to select that strategy which is the maximum among the minima, hence the name - maximin[1]. This criterion is usually associated with the name of Abraham Wald. Mathematically, the choice of action can be expressed as:

$$\max_j \left\{ \min_i Q_{ij} \right\}$$

where Q_{ij} is the payoff resulting from A_i and E_j.

In the example matrix, the conservative decision maker will elect A_2 as his course of action.

If the decision maker is an extreme optimist or an inveterate gambler, he will select the row containing the largest payoff, i.e.,

[1] It is interesting to note that in a two-person zero sum game involving a saddle point, this is the best strategy to play.

A_3. This strategy has been labeled maximax, (maximum maximorum) by the game theorists.

In 1951, Hurwicz attempted a generalization of the maximum criterion to make it less pessimistic. This resulted in a possible compromise between the extreme pessimist and the extreme optimist. Hurwicz suggested an index of optimism[1], α, which is some preselected number between 0 and 1. This means that the decision maker would determine $\max_j \left\{\alpha \max_i Q_{ij} + (1-\alpha) \min_i Q_{ij}\right\}$. If $\alpha = 0$, Hurwicz criterion reduces to the maximin criterion; and if $\alpha = 1$, the criterion becomes maximization of maximum gain. If $\alpha = \frac{1}{2}$, the Hurwicz rule, in the example matrix, leads to the choice of A_3. Because Hurwicz's criterion only takes into consideration the worst and the best for each action, it is not too appealing from a normative point of view.[2] However, this criterion may have some merit because it illustrates how business men make their decisions.[3]

Another criterion, called the "Principle of Insufficient Reason" is usually associated with the names of Thomas Bayes and P. S. Laplace.[4] This criterion attempts to change uncertainty into risk. The argument is that if we are really ignorant of the probabilities with which each state of nature will occur, then we should assume that all of them are equally

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[1] Some writers refer instead to a degree of pessimism.

[2] Good definitions and examples of normative and descriptive theories are given by William J. Morris, (21, Chapt. 13).

[3] For illustrations of a descriptive approach to economic decision making, refer to (26, Chapt. 2).

[4] Bayes originally suggested the criterion but Laplace was responsible for the first extensive use of it. (Lancelot Hogben, Statistical Theory, G. Allen and Unwin Ltd., 1957, p. 21). However, Luce and Raiffa in (16, p. 284) state that Jacob Bernoulli, (1654-1705), first formulated the "Principle of Insufficient Reason".

likely; i.e.,

$$P(E_j) = \frac{1}{n} \text{ for all } j$$

"This principle is extremely vague, and its indiscriminate use has led to many nonsensical results. Writers since Bernoulli's time have attempted to add qualification to the principle and to specify limited interpretation so as to avoid some of the more blatant contradictions", (16, p. 284).

The last[1] criterion which will be considered is the Regret Criterion proposed by Leonard J. Savage (25). When a decision maker employs this criterion, Savage suggests that he really wants to minimize the regret he will experience after he has actually received the specific payoff resulting from his selection of a strategy.

A regret matrix[2] can be computed from the original payoff matrix by subtracting each element from the largest element in its column.

Thus:

	E_1	E_2	E_3	E_4
A_1	0	2	1	0
A_2	2	4	0	0
A_3	3	0	1	3
A_4	1	1	1	3

The decision maker selects the action; i.e., row in which the maximal element is the smallest. This leads to a choice of A_1. Chernoff in 1954

[1] See Luce and Raffia, Chapter 13 for a description of other criteria.

[2] This is equivalent to the opportunity cost matrix calculated from a given payoff matrix in certain inventory problems.

has adequately criticized this criterion and its several drawbacks. Luce and Raiffa (16, pp. 281-2) give a two-page summary of these comments.

The decision criteria which have been considered have given three different results. This is a disturbing state of affairs and would force one to seek a more scientific and systematic approach to the problem of decision making under "pure" uncertainty if possible. Critical analysis of the short-comings of each criterion have been given by many investigators[1] and will not be included here.

If a "pure" uncertainty were an acknowledged reality in the business world, then it would "pay" to continue investigations along these lines. One could then seek to modify or convolute in some fashion the criteria just described. But, before wasting our energies in this direction, the reality of "pure" uncertainty in the business world must be established. "Pure" uncertainty means total ignorance. But, a "feeling" for the future of experienced executives and forecasts based upon statistical studies give a "validity"[2] to subjective probability and lotteries. This permits a quantification of the judgment of the responsible decision maker concerning what the facts may be. "Subjective probability is axiomatized by properties somewhat similar to those of objective probabilities, but it is then shown that subjective probability possesses the defining

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[1] Milnor, J., in Games Against Nature, John Wiley & Sons, 1954, details 10 axioms which he things a good decision rule ought to satisfy. None of the criteria we have considered thus far satisfy all 10 axioms.

[2] There are two schools of thought on subjective probability. One admits its usefulness and validity; the other refuses to recognize it. The latter appear to be in a minority.

property of a subjective (Fechnerian) sensation[1] scale as the term is used in psychophysics", (16, p. 36). Hence, a further study of subjective probabilities leads directly into a review of the utility. This subject is covered in Chapter V.

2.2 Investment Uncertainty and Forecasts

Capital investments based on capital budgeting involves forecasting. Generally, in order for a firm to select a proper investment alternative, it must make some assumption or some forecasts about future sales. In fact, forecasting may very well be the most important variable to be considered in evaluating alternatives, since the goal of forecasting is to determine just what the future holds in store. This is a frontal attack on total ignorance.

The traditional explanation of the dependence of the capital budget on the economic forecast is as follows:

"In order to decide how much a company should invest or what kinds of assets it should acquire, we need a sales forecast for the firm -- to establish its anticipated level of activity.

But the firm's sales forecast cannot be made without some estimate of what the industry is going to do, and this industry's sales forecast in turn depends in large measure on the predicted level of activity in the economy as a whole.

Q.E.D.--the capital budget of any individual firm has a unique and important relation to the general economy forecast." (43, p. 115)

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[1] Fechnerian sensation can be taken as an equivalent to utility.

In his textbook, (38), Prof. Rufus Wixon discusses this relationship between budgets and forecasts in two chapters and then concludes: "It is evident that the only answer to these questions is that successful forecasting is necessary if a firm is to do satisfactory planning and consequently the right kind of budgeting."

In other words, the firm wants to know what to invest in and how much to invest. This depends among other things, upon future sales, and we can never know in advance just what they will be.

Although forecasting is not an exact science and although "estimates are but estimates",[1] it still allows the decision maker to use subjective judgment, in the form perhaps of subjective probability, to rank a preference among alternatives.[2] In some firms this is all that is done to select the best alternative; i.e., just look at the most promising sales forecast. But, in other firms, sales forecasts or other related forecasts, such as future precent of market, are taken as the starting point for the employment of other indices of relative preference. For the latter group of decision makers, forecasting merely served to reduce the ignorance about the future and to give them a mathematical beginning point. Thus, the starting point is usually an estimate--a bit of uncertainty--but this is the best that can be done. Hence, there is some gamble involved, and it is impossible to make sure that every decision will turn out as planned. The decision maker tries to increase the odds

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1 A reference Prior Sinclair uses in describing budgeting--(29, p. 4).

2 For example, sales forecasts may answer the questions, "Shall we expand?", "What size plant shall we build?", "Shall we invest in more research and development of this product? If so, how much more?".

of success in his favor as much as possible. How he does this is described in the next chapter. All of these methods involve some inherent aspects of uncertainty; e.g., future rates of return, level of business activity, actions of competitors, costs of production, future availability of improved models of machines, etc.

2.3 General Analysis of Risk and Uncertainty In Capital Investments

A general analysis by this author of the omnipresent factors of risk (in the gambler's sense) and uncertainty in capital investments reveals that the sources from which these factors originate are mainly five in number:

a. Lack of Sufficient Numbers of Same Type Investments

   As a general rule, only a few of any particular type of investment have been made in the past, which could serve as a guide for designating a reasonably accurate average return and probability of realizing same. The law of large numbers or the law of averages in such cases is inoperative and totally meaningless.

b. Forecast Errors Due to Ignorance of Interacting Factors

   How well does the forecaster know the interactions of the pertinent factors? For example, in trying to predict the sales potential of a new product, are the interactions between prices, income, taste, and competitive products fully understood? The answer is undoubtedly negative in nature; and yet, the very structure of almost all investment decision making processes is built upon forecasts.

c. Environmental Changes in the Future

Almost all aspects of the businessman's world are dynamic in nature. How well does the decision maker know the future prices and demand of his products? Can a violent change in the market completely destroy any demand for his product? They may attempt to account for these fluctuations or changes through a probability method of weighing future returns. But the accuracy of knowing how the future will change, qualitatively and quantitatively is always open to doubt. The world of capital investments is not the static homogeneous environment of the laws of physics.

d. Personal Bias and Habits of Decision Maker

By nature, decision makers generally belong to the optimistic school or to the pessimistic school. In addition, they may be influenced by personal habits, hunches, and predilections. It is possible that bias and prejudice for or against certain investment proposals can be influential on a subconscious level.

e. Inaccuracies Attached to the Analysis Method

Each method of analysis to be examined in Chapter III has certain built-in assumptions about risk and uncertainty. In fact, most of the common methods avoid treating risk or else regard risk for all alternatives as equal. The completeness of proposal details, equally careful treatment of all proposal items, accuracy of these item estimates, etc., may be open to doubt and thus generate uncertainty and risk.

Concerned persons, faced with these foregoing risks and uncertainties, generally attempt to counteract or at least to mitigate this effect in their estimates by turning to probability analysis and when necessary to subjective probabilities. It is these estimates which are usually the starting points for the techniques described in Chapter III.

As a rule, top management is given various estimates; e.g., projected size of market for a new product, costs of production, income from the new product, effects of competition on income, etc. from different departments or individuals. The final decision maker(s) must integrate or convolute those various estimates to arrive at a conclusive yea or nay for the proposal. It is imperative that if a meaningful decision is to be the end product of all estimate integration, that these estimates be made using a consistent and uniform basis. However, it is the opinion of this writer that some confusion and error does exist on an operational level because of differences in estimation philosophy. Probably, the chief sources of such error stems from:

(a) The confusion between expected (average) and most likely estimate. Many individuals, responsible for generating estimates fail to inform the recipient of this information as to what their estimate really means. For example, assume a firm is making plans to modify one of its major products and is interested in determining this effect on gross sales. The following data has been estimated:

	Probability of Occurrence	Income From Sales	Final Weighed Effect
(1) Size of Market Continues Unchanged	.25	$150,000	$37,500
(2) Size of Market Increases	.50	225,000	112,500
(3) Size of Marked Decreases, due to Possible Introduction of better product and/or modification from competition	.25	50,000	12,500

Expected (Average) Sales = $162,500

An estimate for this event can be the most probable result which is that the size of the market increases with an income of $112,500 or it can be that the overall expected sales is $162,500.[1] The entire analysis of profitability could be based on either one. Unless the estimator informs management which estimation base he used, he is adding error, at the very beginning, to whatever evaluation process must employ his estimate. It is easy to see how such error is compounded when different estimators, using different bases, give their findings to the decision making body.

(b) The equal treatment of subjective probabilities when the estimator's confidence are markedly different for each estimate. No doubt, the best form of estimation involves objective probabilities. However, in the face of risk and uncertainty, it is often necessary to use subjective probabilities based on experience and intuition. However,

[1] It is possible to view this as the use of the mode versus the mean for estimating.

it is often found that the estimator is much more confident of some estimates than others. This can be due to more experience in one area than another or the greater accumulation of data on one phase than on others. If the example in "a" above is used, and the estimator claims that "3" is almost pure guess work, but he is a lot surer and more confident of the other two projections; we should, but have not, reflected this difference. In fact, the use of the average weight assumes that all states are given with equal confidence; which is not realistic in many cases. This causes us to consider the next potential source of error.

(c) The transfer of uncertainty to other estimates when subjective probability is used. By its very nature, subjective probability has inherent uncertainties. The estimator usually feels that he is not so completely ignorant of the situation that he should use the Laplace principle of insufficient reason, but at the same time he is not so expertly informed. He usually has some prior information or prior belief about the probabilities in question.[1] But, because the sum of the

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[1] The most ideal situation would be one in which the estimator could actually plot his prior distribution (f(p) vs p) with sufficient approximation. Mathematically, the ideal convolution would involve a prior distribution to represent what he believes, a utility function to represent what he wants, and a distribution to represent what he knows.

probabilities must add up to one, the uncertainties peculiar to one probability must be reflected in the remaining probabilities. To return to the example in "a". If the estimator is still unsure of "3" but he claims that "2" is twice as probable as "1"; then, depending upon what probability is assigned to "3", he has automatically determined "1" and "2" probabilities. In effect, what the estimator has done, is help to distribute his greater uncertainty from one projection to all projections within the system.

(d) The difference in meaning, the "most optimistic" and "most pessimistic" estimates have for different people. These terms are not as definite and universal as is generally supposed, but rather have some vagueness incorporated within their usage. For example, if an estimator looks at the following table:

Projection Effect		Probability of Occurrence
A	=	15^{o}
B	=	14
C	=	14
D	=	14
E	=	14
F	=	14.5
G	=	14.5

Is he to say that the most likely project effect is A and totally ignore the six other equally probable effects? What does most likely mean in this case? Similarly, when the estimator gives a value for the "most pessimistic" effect, what does he mean? Is he stating that under no conditions would he expect anything worse to occur, or merely that in one out of a hundred such situations would he expect the results to occur. Confusion increases when terms such as "moderately optimistic" or "reasonably pessimistic" are employed by the estimators or decision makers in place of the "most pessimistic" or "most likely".

The question which must be answered now is: Assuming that all of the above inconsistencies, confusion and error do exist, is there an operational solution? This author believes that the use of confidence limits or levels may be the answer. The work by Hess and Quigley (59), appears to bear this assumption out. The use of confidence intervals involves statements or tabulations which represent estimates as : "Estimated chance of being Achieved or Exceeded" or "Estimated Chance of being Equal to or Less Than". The first could apply to anticipated sales and the second to expected costs for a new product. The ability to convolute these limits helps to eliminate even more confusion and is a major advantage of this technique. For example, if a decision maker is trying to estimate the profitability of a project and is given the following data:

Anticipated Sales of New Product	Estimated Chance of Being Achieved or Exceeded
$80,000	90%
73,000	70%
60,000	40%
40,000	15%

and Projected Gross Cost	Estimated Chance of Being Equal to Less Than
$65,000	90%
61,000	70%
59,000	50%
46,000	38%

If he treates the confidence levels as probabilities, then the sum or difference of their estimates has at least the same confidence level as is given by the product of the two confidence levels of which it is composed. Hence, an expected profitability of ($80,000-$65,000) = $15,000 = (.9)(.9) = 0.81. Also: (Some Amount Less than 80,000) - (Some Amount Above $65,000) = (.10)(.10) = .01.

Several other convolutions can be evolved. Such manipulations and presentation to a decision maker are much more meaningful than the single expected value for the average profit. Ideally, this author believes, that probability distributions, for all factors are the most meaningful. The computer can be used to convolute these distributions or a manual method employing Monte Carlo techniques can be used on the cumulative distributions for each factor. These distributions incorporate the critical risk dimension and thus allow management to make a quantitative assessment of these involved risks in evaluating a particular investment. The chances of achieving a minimum desired return or of losing money can then be determined.

CHAPTER III CRITERIA FOR EVALUATING INVESTMENTS

3.1 General

In spite of the fact that the survival of a firm may depend upon its investment decisions, "the procedures used to help management make investment decisions are frequently almost unbelievably primitive" (4, p. 2). All available alternatives are seldom given careful analysis. In fact, "even when there is an investigation of alternatives, the information obtained is generally not used in a manner likely to lead to effective final decision", (4, p. 2).

Prof. Donald Istvan, after conducting personal interviews of 48 major companies on capital budgeting problems, stated,

> "Two striking conclusions are to be drawn from this study. The first is that there has not been extensive adoption among the firms studied of the theoretically superior techniques of capital expenditure analysis. This is especially true in the area of economic evaluation. The second is that this investigation has failed to disclose any basic agreement among the managements of the companies regarding the degree of concern and effort that should be expended on the development and implementation of a capital expenditure decision making process. There is not even any general agreement among the number firms of a given industry", (14, p. 97).[1]

These comments should serve as a stimulus to investigate in depth the more generally used methods of evaluating investment proposals. What are the merits and shortcomings of each? Why is it that each method usually gives different ranking of investment alternatives? It behooves management today, because of the very nature of today's economic business environment, to acquaint itself with the various

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[1] Prof. Istvan then proceeds to give, in detail, what he believes are the reasons for this situation.

methods and to employ the best.

Most of the methods to be reviewed have the following characteristics in common:

a. An estimate of the firm's cost of capital is required.

b. Estimates are needed of the incremental cash flows anticipated from a proposal over time.

c. Estimates are required for the incremental cash outlays required to implement each proposal.

d. The overall, basic goal employed is profit-maximization.

e. The risks involved or the future make-up of the assets and liabilities that will be determined in part by the investment decisions currently being made, are not considered.

It has been agreed that because a final selection by management is seldom made on the basis of such economic analysis, the method used is inconsequential. This may be true if the executive(s) making the final decision is (are) acquainted with all the pertinent uncertainties and problem areas. But, as a general rule, in the larger firm, the executives responsible for the decision making are fed the data by some subordinate group. In these cases, the method used to evaluate alternatives may be very important, since the various techniques in common use today give different ranking to identical sets of investment proposals. "Substantial improvements in efficiency and income may result if a more adequate measure can be discovered and widely adopted", (4, p. 10).

3.2 Payback Period

In 1954, Joel Dean referred to the payback period method as "unquestionably the most widely used measure of investment worth", (47, p. 121). In 1961, Prof. Donald F. Istvan found that 34 out of the 48 major firms he interviewed "do make use of the payback measure of acceptability", (14, p. 91). Although recent articles and textbooks on capital budgeting are quick to point out the weaknesses of this method, "the payback period remains one of the simplest and apparently one of the most frequently used methods of measuring the economic value of an investment", (4, p. 17).

A proposal's payoff period is defined as the ratio of its cost to its annual earnings before depreciation.[1] If an investment is expected to produce a stream of cash income that is constant from year to year, then the payback period can be determined by dividing the total original cash outlay by the amount of the annual cash anticipated income. But, if the anticipated income is not constant from year to year, then the payback period is determined by adding up the income expected in successive years until the total is equal to the original outlay.

The proposal's rate of profit, r, with reference to payback, can be calculated as follows, where:

C = cost of a given alternative or proposal.

E_t = earnings before depreciation that the alternative is expected to provide in year t.

[1] Prof. Barish defines "the payoff period as the time required for an investment to pay for itself through the net operating advantage or revenue which would result from its installation". (3, p. 387)

n = the number of years the equipment is expected to last.

S_n = the scrap value of the equipment at t = n.

r = the alternative's expected rate of profit.

Using the standard discount formula:

$$C = \frac{E_1}{1+r} + \frac{E_2}{(1+r)^2} + \ldots + \frac{E_t}{(1+r)^t} + \frac{E_n+S_n}{(1+r)^n}$$

Assuming that $E_1 = E_2 = E_3 = E_n$, and that $\frac{S_n}{(1+r)^n}$ is small in relation to C, we obtain the simplified formula:

$$C = \sum_{t=1}^{n} \frac{E}{(1+r)^t} = \frac{E}{r}\left[1- \left(\frac{1}{1+r}\right)^n\right].$$

Finally,

$$r = \frac{E}{C} - \frac{E}{C}\left[\frac{1}{1+r}\right]^n$$

When the planning period is sufficiently large, that is, n is a large number, the above formula simplifies into

$$r = E/C$$

But, E/C is the reciprocal of the payoff period. Therefore, it is noted that:

a. If the proposal will earn the same amount forever, its rate of return is simply the reciprocal of its payoff.

b. If the proposal will last a finite number of years, the rate of return, r, is at most the reciprocal of the payoff period and smaller than this by the amount equal to $\frac{E}{C}\left[\frac{1}{1+r}\right]^n$

Thus, we have a relationship between the payoff period and a proposal's rate of return. The margin of error, $\frac{E}{C}\left[\frac{1}{1+r}\right]^{n}$, decreases as the life of the alternative increases. For example, "a statement such as 'This investment will be paid out in three years' is equivalent in many companies to a 33% rate of return, either by direct calculation or by implication". (60, p. 35)

Prof. Barish (3, p. 388) states, "Let us note that the payoff period is another interpretation that can be given to the present value factors for annuities". But, he does not give the required conditions; i.e., assuming that an investment with a return equal to the cost of capital of the firm earns equal income each year, the maximum payback period is equal to the present value of a dollar per period for n periods discounted on the basis of the cost of capital. For example, if a machine has an economic life of 15 years and the rate of return is 10%, the present worth table for $1 for 15 years at 10% gives 7.606. This indicates that the maximum capital recovery takes place in 7.6 years:

$$E \times P^{r}_{n} = C$$

$$\text{Payback Period} = \frac{C}{E}$$

$$\text{Payback Period} = P^{r}_{n}$$

where P^{r}_{n} = present value of a dollar per period for n periods discounted at an r rate of interest.

C = cost of a proposal

E = earnings before depreciation

"Because of the limiting assumptions, especially equal annual proceeds, this formula is not useful in making decisions, but it is useful in showing the absurdity of certain payback conventions." (4, p. 33)

Basically, the cash payback period index has two weaknesses:

a. It fails to give any consideration to income earned after the payback date.

b. It fails to take into account the differences in the timing of proceeds earned prior to the payback date.

These weaknesses can easily be illustrated by use of the following table:

TABLE I

Investment Costs and Income to Show Weaknesses of Payback Method

		INCOME				
Investment	Initial Cost	Year 1	Year 2	Year 3	Payback Period	Rank
A	10,000	10,000			1	1
B	10,000	5,000	5,000	5,000	2	2
C	10,000	2,000	4,000	12,000	2 1/3	3
D	10,000	10,000	3,000	3,000	1	1
E	10,000	6,000	4,000	5,000	2	2

Although both investments A and D are ranked as 1, D is definitely better. Any useful investment index must make such a difference known. Also, B and E are ranked the same, but investment E is superior to investment B since the only difference between both investments is the $1,000 of investment E in Year 1, which is available one year earlier.

Joel Dean prefers to list the weaknesses of this method as follows:

a. "Payback tends to overweigh the importance of liquidity as a goal of the capital-expenditure program.

b. "It ignores capital wastage. By confining analysis to the project's gross earnings it takes no cognizance of its probable economic life.

c. "It fails to consider the earnings of a project after the initial outlay has been paid back." (47, p. 24)

When Prof. Istvan's statement: "Even where correctly employed, that is as a measure of liquidity, complications in calculations can arise because, just as with the simple rate of return, investment can be considered as initial, average, gross, or net of salvage value. Advantage can be before or after financing costs, capital consumption, and income taxes", (14, p. 88), is investigated mathematically, certain interesting relationships evolve. For instance, consider:

a. Average Investment:

A formula involving average investment return can be written as:

$$r = \frac{E-D}{\frac{1}{2}C}$$

where E = annual equal earnings before depreciation

D = yearly depreciation amount

C = cost of the alternative

r = annual return rate

n = life of project

But, D = C/n because annual depreciation is equal to the cost divided by the life of the proposal's equipment.

Therefore, by substitution,

$$r = \frac{2E}{C} - \frac{2}{n}$$

And,

$$r = \frac{2E}{C}$$

as $n \to \infty$

Which means that as the project's life extends far into the future, the rate of return approaches twice the reciprocal of the payoff period. Accordingly, while the error term, which we noted earlier in the general formula disappeared, the error in the average investment becomes E/C and r = (2) x (E/C). To help compensate for this error, it has been suggested that the full cost be used:

$$r_t = \frac{E-D}{C} = \frac{E}{C} - \frac{1}{n} .$$

This tends to give more conservative results.

b. After-Tax Rate of Return:

Let I = D + (E-D)(1-i)

$$= \frac{C}{n} + (E - \frac{C}{n} - Ei + \frac{C}{n} i)$$

$$= E(1-i) + \frac{iC}{n}$$

where, i = income tax rate, I = annual income before depreciation and after tax
D = depreciation amount, assuming straight line depreciation = C/n

If we assume i = 0.50, we have:

$$I = \frac{E}{2} + \frac{C}{2n} .$$

If we substitute this value of I, for E, the pre-tax income, in the original rate of return equation, $r = E/C - E/C \left[\frac{1}{1+r}\right]^n$, we have:

$$r' = \frac{E}{2C} + \frac{1}{2n} - \left[\frac{E}{2C} + \frac{1}{2n}\right] x \left[\frac{1}{1+r}\right]^n$$

Hence, the after-tax rate of return approaches one-half the pre-tax rate as n becomes larger. Myron Gordon has shown that, "for n equal to the payoff period the error in the payoff reciprocal is not small, but for n 50% greater, the error is negligible. The error remains negligible for all values of n greater than this value." (55, p. 53)

In favor of the payback method, Prof. Istvan states, "As a measure of capital recovery, a correctly calculated payback figure[1] is unrivaled. The greatest virtue of the payback measure is its extreme simplicity". (14, p. 90) A manual in evaluating investment proposals for the Imperial Oil Limited[2] claims, "The Payback Method is not common as a measure of the profitability of an investment, and it is satisfactory to determine whether a proposal is extremely attractive or not worth further study. However, it is

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[1] A correctly calculated payback measure employs:
- a. Net operating advantage before depreciation allowance.
- b. Net operating advantage on a year by year basis rather than any type of average advantage.
- c. Initial investments before salvage recovery on the proposal but after salvage on any replaced facilities.

[2] The Discount Method of Evaluating Investment Proposals, Marketing Research, Imperial Oil Limited, Toronto, Canada, April 12, 1955.

inadequate as a final judge of the majority of projects lying between these extremes."

Joel Dean agrees with the above when he states, "Payback can serve as a coarse screen to pick out high-profit projects that are so clearly desirable as to require no refined rate-of-return estimates and to reject quickly those projects which show such poor promise that they do not merit thorough economic analysis. In addition, it may be adequate as a measure of investment worth for companies with a high outside cost of capital and severely limited internal cash-generating ability in comparison with the volume of highly profitable internal investment opportunities." (47, p. 123)

3.3 Rate of Return

3.31 External-Present Value Method

A sequence of returns E_0, E_1, $E_2 \ldots E_n$ is said to have at present the discounted value of:

$$I = E_0 + \frac{E_1}{1+r_1} + \frac{E_2}{(1+r_1)(1+r_2)} + \ldots + \frac{E_n}{(1+r_1)(1+r_2)\ldots(1+r_n)}$$

where r_i is the annual interest (discount)[1] rate. Discounting[2] calculations are usually simplified by assuming that the interest rate does

1 "A borrower and a lender agree on an interest rate, while a decision maker uses a discount rate." (18, p. 14)

2 "Discounting is characterized by the fact that it proceeds from future to present, inversely to the passage of passage of time. Physically the stream of services flow parallel with time, beginning with the asset invested." (18, p. 11)

not vary with time. Then,

$$I = E_0 + \frac{E_1}{1+r} + \frac{E_2}{(1+r)^2} + \ldots + \frac{E_n}{(1+r)^n}$$

The conventional investments are characterized by having one or more periods of outlays (or income) followed by one or more periods of cash income (or outlays). The nonconventional investments have one or more periods of outlays (income) interspersed with periods of positive (negative) cash flows. The pattern is important when one attempts to calculate an internal rate of discount since the nonconventional investment tends to give rise to multiple rates.

If one employs continuous rather than discrete discounting, the present worth of a cash stream can be written as:

$$I = \int_0^T E(t)\, e^{-(jt)}\, dt$$

where T is the life of the project, E(t) is a continuous return function rather than the sequence of returns E_1, $E_2, \ldots E_2, \ldots E_n$, and $e^j = 1+r$.

The employment of either discrete or continuous functions has presented a problem to some investors. In fact, even when one employs discrete calculations, the problem of annual, semi-annual, or quarterly compounding has been raised as an issue. Actually many of the major financial outlays from a business occur at discrete intervals; e.g., quarterly taxes, quarterly dividends, monthly salaries, semi-annual interest, etc. However, many of the financial inflows occur somewhat continuously. If this is so, "no particular compounding interval is exactly appropriate." (30, p. 132)

Actually, the difference between annual discrete compounding as against continuous compounding is small. As rates and time period become higher, the relative difference in present values becomes significant. "But, because the present values of distant sums fall very sharply at high rates of discount, the absolute difference in final results obtained in solving for rates of returns is minor." (30, p. 132) The following table and graph will serve to illustrate these points:

TABLE II

DIFFERENCE BETWEEN CONTINUOUS (C) AND ANNUAL (A) DISCOUNTING INTERVALS. PRESENT VALUE OF ONE DOLLAR DUE AT END OF N YEARS.

	5 percent		10 percent		15 percent		20 percent	
N	C	A	C	A	C	A	C	A
1	.9512	.9524	.9048	.9091	.8607	.8696	.8187	.8333
5	.7788	.7835	.6065	.6209	.4724	.4972	.3679	.4019
10	.6065	.6139	.3679	.3855	.2231	.2472	.1353	.1615
15	.4724	.4810	.2231	.2394	.1054	.1229	.0498	.0649
20	.3679	.3769	.1353	.1486	.0498	.0611	.0183	.0261
25	.2865	.2953	.0821	.0923	.0235	.0304	.0067	.0105

R. M. Adelson in a footnote does not seem to agree completely: "This distinction (between continuous and discrete discounting) does not affect the arguments here but it is worth noting that it can make considerable difference to certain numerical results", (39, p. 20). But, Barish states, "In performing interest calculations, there is no real advantage to using continuous compounding", (3, p. 64). Pierre Massé believes the only advantages, if any, of using continuous over discrete discounting are theoretical since, "continuous discount rate, permits a more concise notation and a more direct use of the advantages of the differential and integral calculus", (18, p. 16).

The present value method, by reducing a sequence of future returns to a single number or index, allows the decision maker to rank order proposals. If two sequences of returns (E_0, E_1, E_2--E_n) and (E_0', E_1', E_2',--E_n') with present values equal to I and I', are such that $I > I'$, the first will be preferable to the second. If $I = I'$, they are equivalent from the standpoint of present worth only. However, most writers seem to employ net present value as their criteria because it allows one the immediate advantage of accepting or rejecting any proposal and then allows for proper ranking of acceptable proposals. This method is executed as follows:

a. Choose an appropriate rate of interest.

b. Compute the present value of the cash flows expected from the investment.

c. Compute the present value of the cash outlay required by the investment.

d. The present value of the inflow minus the present value of the outflow is the net present value of the investment.

e. Accept all independent investments whose present value is greater than or equal to zero. Reject all investments whose present value is less than zero.

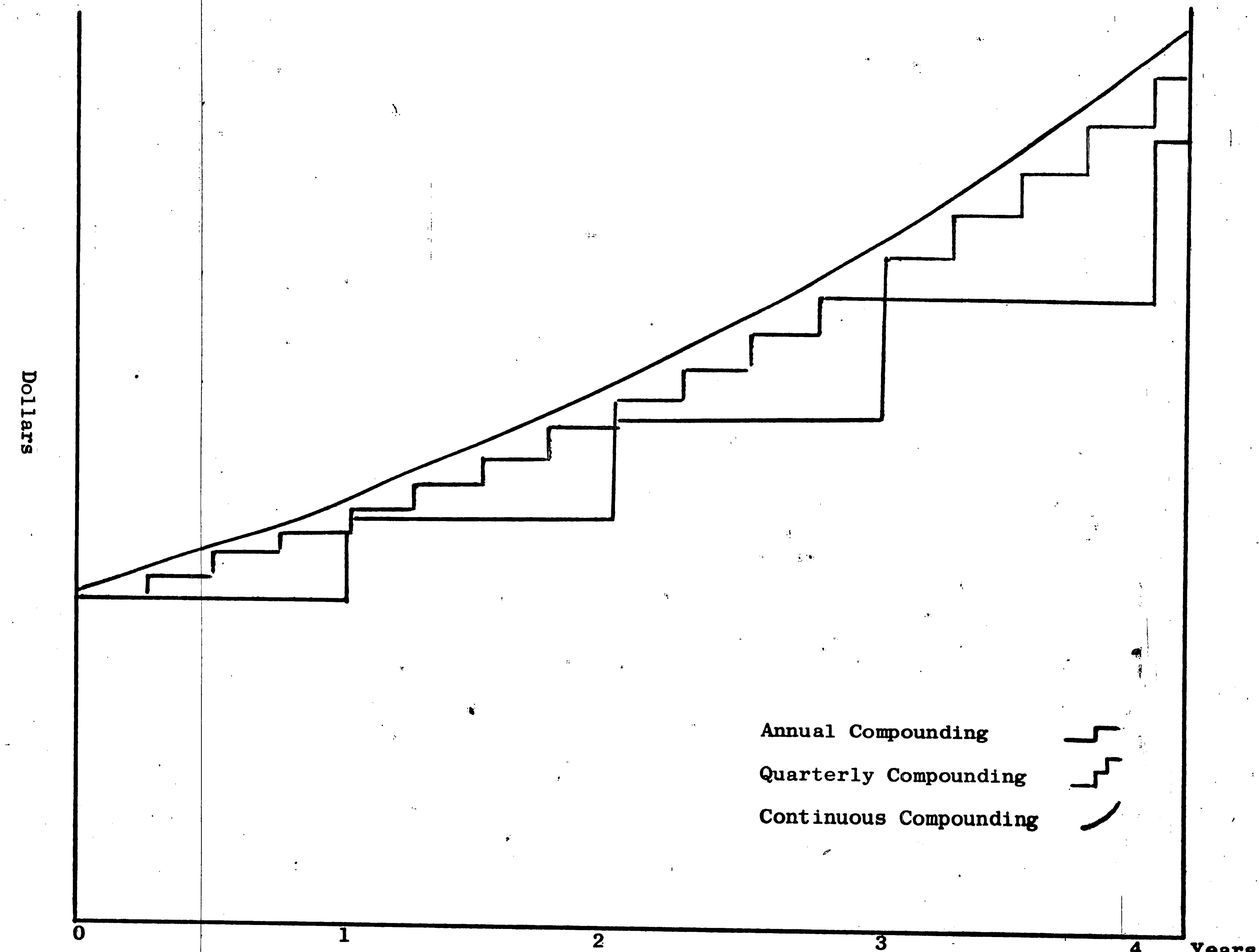

FIGURE 1

Discrete and Continuous Compounding

f. Rank order all acceptable proposals on basis of magnitude of present value.

The choice of the proper discount rate can affect either the accept/reject criterion or the relative ranking or both. How does one decide upon the proper discount rate? Is it the cost of capital? Is it a fixed rate that management has instituted regardless of the fluctuating marketing to take into account the long range effect of the proposal? Is it meant to include a buffer factor for uncertainty? Is it meant to take into consideration the cash requirements of the owners? Is the rate based on the type of investment; i.e., independent, mutually exclusive, complementary, or substitute? There cannot be one correct answer since no two firms are identical in resources, investment potential, financial set-ups, ability to handle uncertainty, reasons for investment, and sources of funds. Accordingly, "it should be appreciated that computations using the present-value method are indications rather than numbers with 100% certainty and accuracy. It should be stressed that any decision about investments must in the last analysis be based on as complete a consideration of all the relevant factors as it is possible to provide, and that the probable present value of an investment proposal is only one factor although a very significant one, that must be considered in arriving at a final decision", (4, p. 101).

Although some writers give different reasons for using the present value method, only one is really valid; i.e., to determine whether the investment yields more income than alternative uses of the same amount of money. Uncertainty and personalized time preferences of investments are not valid enough reasons for the employment of this technique. If a decision-maker uses the present value technique to discount for uncertainty, he is assuming that the probability of a dollar of estimated cash actually materializing will decrease each year by a fixed percentage of the probability in the preceding year. Of course, the amount of percentage decrease each year depends upon the rate of interest. "In particular, the application of present value discount factors to preliminary estimates of expected cash flows will not ordinarily be an accepted way of adjusting for risk." (4, p. 55). It must be noted that if, in "acceptable" cases, the present worth method is used to account for uncertainty, then, it must be applied a second time to account for time differences in the value of money.

"Since discounting, as generally defined, is truly relevant only to situations of perfect liquidity and no uncertainty, it is not surprising to find that most attempts to incorporate risk into these criteria have resulted in considerable confusion." (39, p. 23) If the decision-maker does not pretend that uncertainty does not exist and if he does not merely use his intuition, what are some of the recommended techniques to incorporate uncertainty into the present value method? (The frequently stated procedures of shortening the expected life of an asset, estimating earnings very conservatively or using higher discounting rates for riskier proposals are not included in this group.)

a. Method 1 - Recommended by Profs. Bierman and Smidt, (4, Chapt. 9).

1. Determine the present value of net cash flows for three different series of events:

(a) Most Probable

(b) Reasonably Pessimistic

(c) Reasonably Optimistic

2. Weigh the three present values, using the best information or using standard weights such as 50% for most probable and 25% for optimistic predictions.

3. The sum of the three weighted present values may be used to represent the present value of the investment taking uncertainty into consideration.

b. Method 2 - As presented by Prof. Robert Schlaifer, (26, Chapt. 2).

This modus operandi is actually similar to Method I, but it assigns values to outcomes according to their utility rather than their present values. The probability of each outcome is also accounted for in the product of utility and probability; i.e., expected utility. Of course, the determination of corporation utility values may not be readily determined.

c. Method 3 - Joel Dean's Probability Multiplier.

A probability of occurrence factor, ($0 < \text{factor} \leq 1$), relative to the estimated earnings for each project for each year is employed. The probability factor is smaller for more distant years to reflect greater uncertainty. "This method is capable of fine discrimination among years as well as among projects. It is particularly appropriate when the whole return from the investment becomes sharply more uncertain at some future date", (9, p. 31).

Actually, the search for better decision making in the face of uncertainty has generated other models; e.g., dynamic and stochastic programming. "This area (of uncertainty consideration) is still in its infancy, and there is little that can presently be brought effectively into the investment decision of a complex corporation." (4, p. 132)

3.3 Rate of Return

3.32 Internal--Discounted Cash Flow Method[1]

Inherent in this method is the present value concept, but without the choice of an "arbitrary" rate of interest. The basic procedure is to find a rate of interest that will make the present value of the cash income expected from an investment equal to the present value of the cash outlays required by the investment. By a process of trial and error, the approximately correct rate of interest can be determined. This method is identical to the technique long employed by financial circles for computing the expected rate of yield to maturity of a bond which is purchased for an amount different from its face value. For example, a 5% $1,000 bond, which matures in 5 years, and has a net purchase price of $1.044.52 will yield 4%.

[1] Other terms used to define this technique are internal rate of return on investment, present value return on investment, profitability index, investor's method, yield of an investment method, Keynes' marginal efficiency of capital method, and the scientific method.

A main appeal of the internal rate of return method "is that the internal rate of return for a proposed project may be computed without a knowledge of the market rate of interest", (44, p. 20). But, according to Pierre Massé, "if one takes time to think the matter over, this apparent facility is only an evasion", (18, p. 23). Profs. Bierman and Smidt hold the same opinion: "The yield of an investment must be compared with the cost of capital. The cost of capital is no less important to yield (i.e., internal rate of return method) than to present value, although it enters at an earlier stage in the computations of the present value method", (4, p. 35). This is especially evident when the decision maker employs a reference or cutoff rate in an accept/reject action of a proposal:

a. First, the decision maker ranks all investment proposals in descending order according to their internal rate of return.

b. Second, he accepts all investments whose profit yield or internal rate of return is as great or greater than the cost of capital.

The relevance of the cost of capital to the internal rate method can also be noted simply by interpreting what is meant by an internal rate of return of a conventional investment: "It represents the highest rate of interest an investor could afford to pay without losing money, if all the funds to finance the investment were borrowed and the loan was repaid by application of cash proceeds from the investment as they were earned." (4, p. 26)

It is not always possible to obtain a unique rate of return for a given proposal. Under certain circumstances, it is possible that a proposal:

a. Will have no rate of return

b. Will have one rate of return

c. Will have more than one rate of return.

All conventional investments (negative cash outflow followed by a string of positive cash inflows) will have one and only one rate of return. But, a non-conventional investment; (an investment which has one or more periods of outlays (income) interspersed with periods of positive (negative) cash flows), may fit into either of the above, "a," "b," or "c" class.

An example of a non-conventional investment with no yield would be an investment having cash income of $200 and $300 in periods 1 and 3, respectively, and cash output of $400 in period 2. Mathematically, we can express this as the equivalent of:

$$300 X^2 - 400 X + 200 = 0$$

where X represents the discount factor. Of course, this equation has no solution in the domain of real numbers. "However, a project for which there is no real value of the internal rate of return is not necessarily a bad one." (44, p. 21)

The problem of multiple rates was first examined by James Lorie and Leonard Savage in 1955 in an article titled, "Three Problems in Rationing Capital", (66). According to these investigators, the types of investments yielding more than one rate of return "are rare, but they do occur, especially in the extractive industries", (66, p. 63). But, Prof. Richard Bernhard disagrees and claims, "it is not difficult to find examples in practical settings for which R_n (the terminal cash flow) is not positive and hence for which the internal rate of return is not unique.[1] For example, the terminal cost of demolishing a building or of disposing of radioactive waste might be such as to make R_n negative", (44, p. 21).

Also, J. Hirshleifer (62), has presented several patterns of net incremental returns which do not lead to a unique value of r. M. Klein does the same in his article (64, p. 69).

An example, due to Ezra Solomon, will serve as an illustration of multiple internal rates of return.

Assume that the proposal being considered is the installation of a larger oil pump that would get a fixed quantity of oil out of the ground more rapidly than the pump that is already in use. Also, assume that by operating the existing pump the investor can expect \$10,000 a year hence and \$10,000 two years hence; and that by installing the larger pump at a net cost of \$1,600 now, he can expect \$20,000 a year hence and nothing the second year. The installation of the larger pump can be viewed as a project having the following cash flow characteristics:

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[1] C. S. Soper in (79, p. 175), has shown that sufficient conditions for r, the internal rate of return, to be unique are:

a. The final net return, E_n, in the life of the project must be positive.

b. $- E_0 > \sum_{s=1}^{s=k} \frac{E_s}{(1 + r)^s} \qquad k = 1, 2, \ldots, n-1$

Time Period	Incremental Cash Flow Due to Investment
t_0	- 1,600
t_1	+10,000
t_2	-10,000

Application of the standard procedures to find the internal rates which make the algebraic sum of the discounted cash outflows and inflows equal to zero, yields two answers, 25% and 400%. Which of these two rates is correct? "Neither of these rates is a measure of investment worth, neither has relevance to the profitability of the project under consideration, and neither, therefore, is correct. The fault lies in the incorrect application of the 'usual prescription' for finding the rate of return", (30, p. 130). To find the correct answer, we must merely answer the question, "What is it worth to the investor to receive $10,000 one year earlier than he would have otherwise received it?" If the investor spends $1,600 at time t_0 for a larger pump, he will have at time t_2 ($10,000)(X%) where X% represents the rate of yield applied to the $10,000. Equating the outlay of $1,600 and the gain of $10,000 (X%) to zero over the three time periods gives the appropriate rate of return. "Using this approach, a unique and meaningful rate of return can always be found for any set of cash inflows and outflows", (30, p. 131).

There are two different ways to analyze multiple returns, graphical and mathematical. Each shall now be briefly examined:

A. Mathematical Analysis of Multiple Rates[1]

Actually, the proof shall be concerned with conditions necessary for a unique rate of return. By not satisfying these singular conditions, it is possible to have multiple returns.

The present value of the income flow can be defined as :

$$V = \int_{o}^{T} E(t)e^{-st}dt$$

where T represents the life of the project and s is the instantaneous rate of interest.

Similarly, the present value of the outflow of investing can be defined as:

$$W = \int_{o}^{T} C(t)e^{-st}dt$$

where C(t) represents the capital investment schedule or rate at which capital is being invested at time t. To calculate an internal rate of return, we set:

$$\int_{o}^{T} E(t)e^{-st}dt = \int_{o}^{T} C(t)e^{-st}dt$$

In general, it is not possible to solve this equation directly for a value of s. However, numerical methods can be devised with the aid of which the equation can be solved. In order to investigate the characteristics of this equation further, it is necessary to define a function U(t,s) as:

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1 The approach which the author uses is similar to the Lefkovits', Konner's, and Harbottle's presentation at the 1959 Fifth World Petroleum Congress (65).

$$U(t,s) = \int_0^t C(x)e^{-sx}dx - \int_0^t E(x)e^{-sx}dx$$

which is the present value at the rate s, of the unpaid capital at time t. Now, if T denotes the life of the project, then $s \geq 0$, is an internal rate of return if

$$U(T,s) = 0$$

Now, consider a net cash flow:

$$F(t) = E(t) - C(t).$$

where

$$\int_0^T F(t)e^{-st}dt = 0$$

defines a rate of return for $s \geq 0$.

Because T is the termination date, $F(T) = 0$ for $t > T$; therefore, we can restate the former equation as:

$$\int_0^\infty F(t)e^{-st}dt = 0$$

This integral represents the Laplace[1] transforms of the function F(t) which can be represented by f(s). Hence, a rate of return of a project is a non-negative root of the equation:

$$f(s) = 0$$

Now, it is possible to state the conditions for a project to have only one rate of return:

Condition 1:

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[1] By analogy to electrical engineering, one can think of the input as the cash outflows, the black box as the financial manipulation of the project involving rates of return, and the output as the cash inflows.

a. $f(o) \geq 0$,

b. $F(t)$ have only one sign change. In words, this means that the total undiscounted income from a project is not smaller than the capital investment into the project, and also the net cash flow does not become negative again after it has become positive.

Condition 2:

a. $U(t,s) \geq 0$ for all $t \geq T$

(It is easier to "see" this relationship if we redefine $U(t,s)$ based upon the definition of $F(t)$:

$$U(t,s) = -\int_0^t F(x)e^{-sx}\,dx$$

where $F(t)$ is the net cash flow rate of the project and s is the rate of return of the project).

This condition states that in order for a project to have a unique rate of return, it is sufficient that the unpaid capital function remain non-negative at all times when a previously determined rate of return is used for discounting. Stated in another way, a project has more than one rate of return if the function $U(t,s)$ changes sign as a function of t. Whenever $U(t,s)$ is positive, it represents the unpaid capital, on which it is assumed that the interest rate, s, is being paid. As $U(t,s)$ decreases toward zero, and then becomes negative, $U(t,s)$ represents a surplus which is being accumulated out of the income, but being used up later in the project.

It is assumed that this surplus is also accumulating interest at the same rate as is being applied to the unpaid capital.

It should be noted that if a project satisfies either one of these two conditions listed above, it will always have a unique rate of return; otherwise, it may have multiple returns.

Proof:

Condition 1 can be derived as a special case of Condition 2, since if a project has only one investment period, then the unpaid capital will always be positive. Accordingly, Condition 2's proof will be given. The approach is to show that if $U(T_2 s_1) = 0$ and $U(t_2 s_1) > 0$, $t < T$, then $U(T_2 s) = 0$ has only one solution $s = s_1$.

Assume that there exists a solution s_2 to $U(T_2 s) = 0$ and $s_2 > s_1$. Then let:

$$\Delta = \int_o^T F(x) \left[e^{-s_1 x} - e^{-s_2 x}\right] dx$$

But since s_2 is a solution of $U(T_2 s) = 0$, $\Delta = 0$. Rewrite:

$$\Delta = \int_o^T F(x) e^{-s_1 x} \left[1 - e^{(s_1 - s_2)x}\right] dx$$

This integrates by parts to:

$$\Delta = \left[1 - e^{(s_1 s_2)x}\right] \int_o^x F(y) e^{-s_1 y} dy \Bigg|_{x = o}^{x = T} +$$

$$\int_o^T (s_1 - s_2) e^{(s_1 - s_2)x} \int_o^x F(y) e^{-s_1 y} dy dx$$

The first term vanishes at both limits and the second term is negative since $U(T_2 s_1) = 0$ and $s_1 < s_2$. Therefore, $\Delta < 0$ and so s_2 cannot be a solution of $U(T_2s) = 0$. The identical procedure is used to show that if $s_2 < s_1$, s_2 is not a solution of $U(T_2s) = 0$.

B. Graphical Method for Analyzing Multiple Rates of Return

For conventional investments, the present value is a steadily decreasing function of the cost of capital.[1] This can be illustrated by means of the following graph:

Net Present Value vs. Rates of Return for Conventional Investments

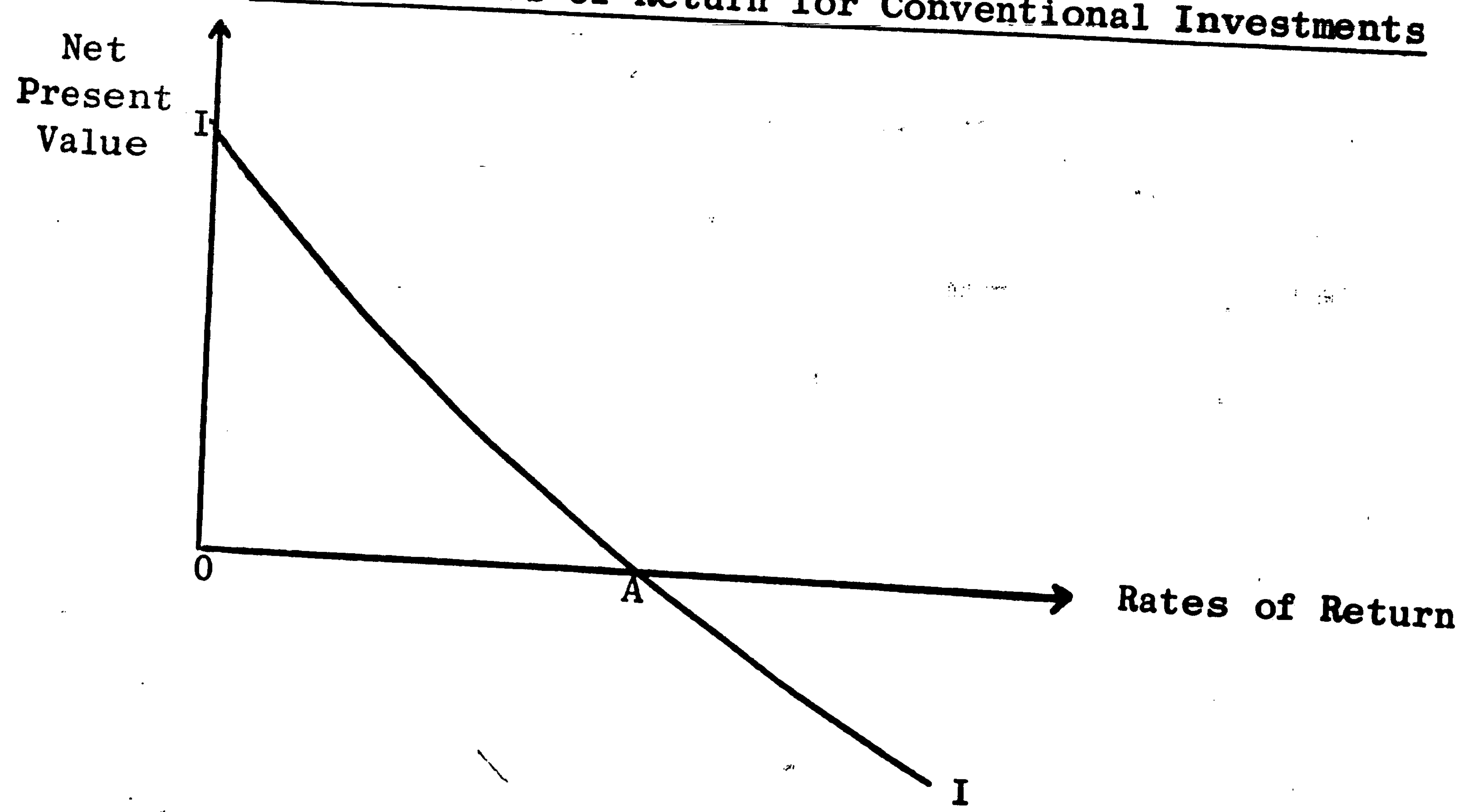


FIGURE 2

The curve I - I represents the net present value of an investment at various rates of interest. 0 A indicates the particular internal rate of return on the given investment, since the yield or return of a cash flow is defined as the rate of interest that makes the net present value zero. Accordingly, the return is the point at which the net

[1] It is easy to visualize this by employing the formula for continuous discounting:

$$I = \int_0^T E(t)e^{-(jt)}$$

As $|j|$ increases, I decreases.

present value line crosses the horizontal axis.

The basic question which has to be answered is: Under what conditions can the net present value of an investment be something other than this steadily decreasing function of the cost of capital? In other words, why do some net present value curves intersect the x axis in more than one place?

To analyze this situation, let's look at a proposal that intersects the "x" axis in two places. The example given on page 51 is such a possibility.

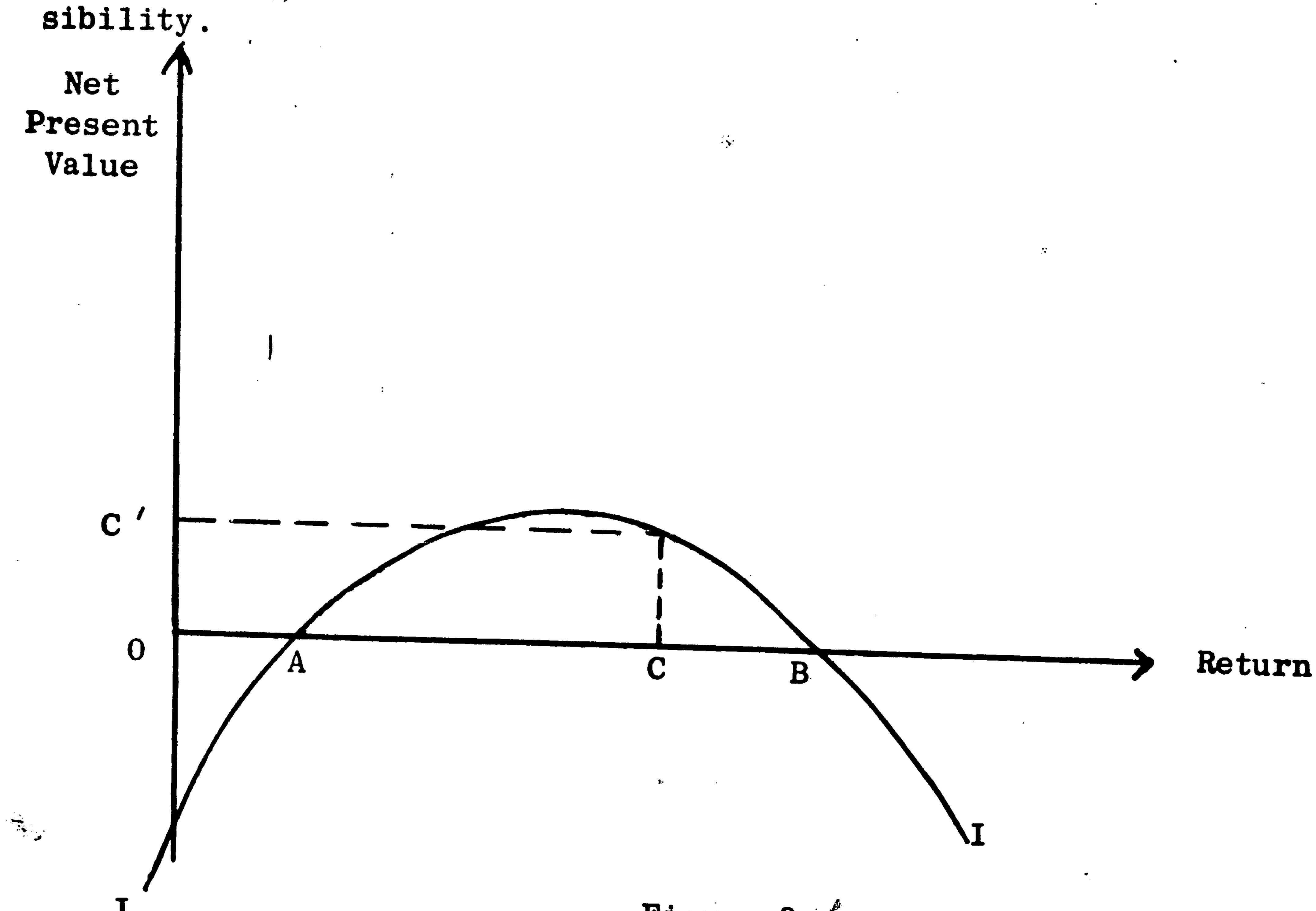


Figure 3

When A Proposal Yields More Than One Rate of Return

In Figure 3, A and B represent the two yields, C is the cost of capital, C' is the net present value of the investment at the firm's cost of capital.

It is noted from Figure 3 that:

a. As the cost of capital approaches zero, the net present value tends to be negative if this net value is negative.

b. As the cost of capital increases, the present value of the final net cash outflows diminishes in importance in relation to earlier flows and this lessening effect causes the net present value of the proposal to become positive. This seems to imply that more than one outflow is involved as in a non-conventional type of investment.

c. As the cost of capital continues to increase, all future cash flows continue to diminish in their importance, causing the present value of the proposal to approach the initial outlay as a limit.

d. The first part of the graph is typical of that of a loan, while the second part has the downward slope typical of an ordinary investment. Figure 4 will serve to illustrate this:

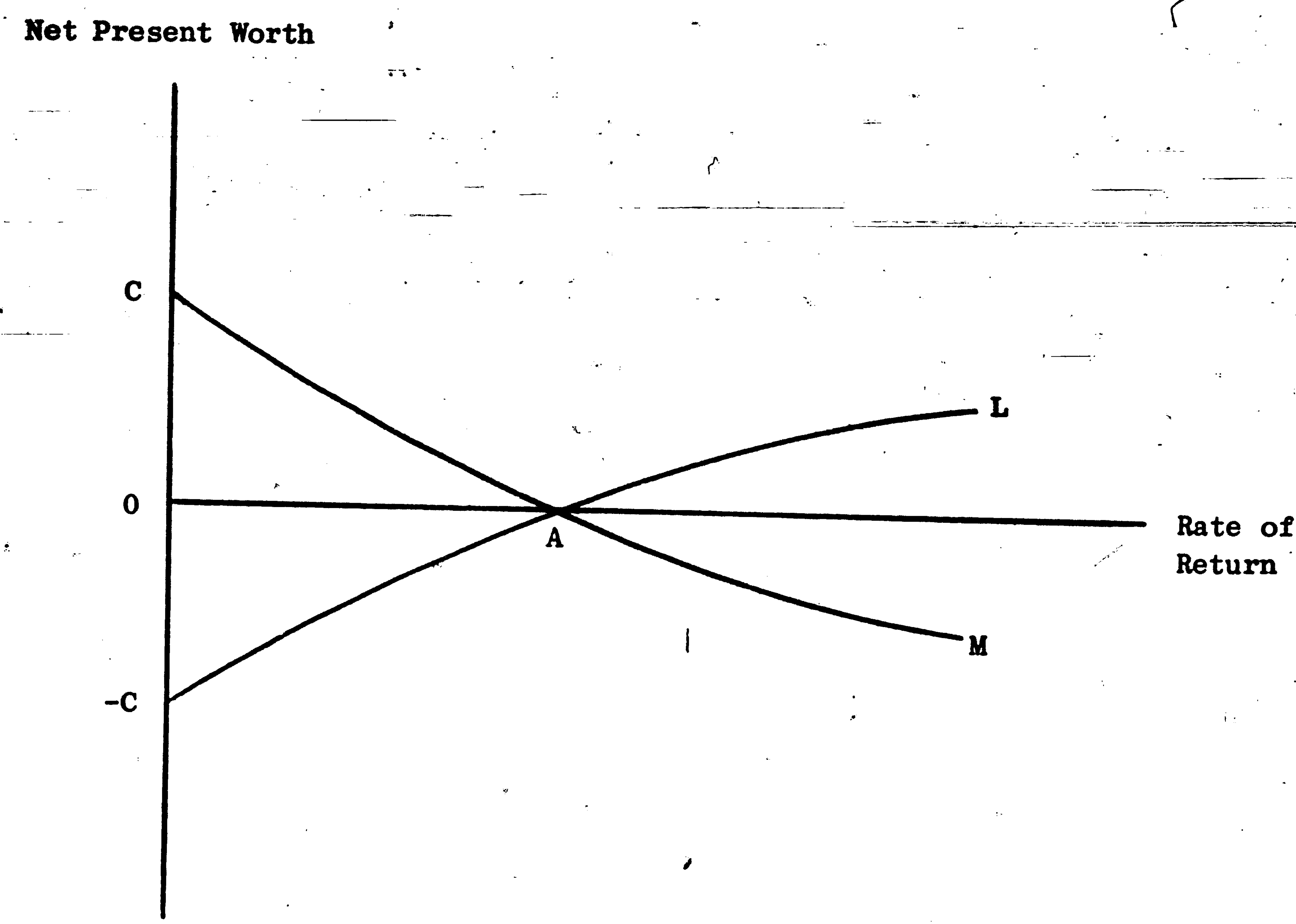


Figure 4

Same Investment As Seen By Lender and Borrower

Curve L: Investment as seen by the lender

Curve M: Same investment as seen by the borrower

Hence, this series of cash flows as shown in Figure 3 would be worthwhile at rates of discount between A% and B% only.[1] The reasoning for this lies in the fact that for the loan type of cash flows, the yield represents the lowest rate of discount at which the net present value would be positive and the borrowing desirable; and for conventional investments, the yield represents the highest rate of discount at which the net present value would be positive and the investment

[1]Since A and B represent the two yields.

desirable. If the cash flows representing the curve in Figure 3 were inverted, i.e., income becomes outlays and vice versa, the resulting cash flows would be desirable only at interest rates that were less than A% or greater than B%.

3.3 Rate of Return

3.33 Discounted Cash Flow vs. Present Value

In an article, Arithmetic of Capital Budgeting Decisions, (78), Ezra Solomon presents the following problem:

Assume that two mutually exclusive projects, X and Y, are to be decided upon. Project X requires an outlay of $100 now, at time t_o and promises to return $120 exactly 1 year hence at time t_1. Project Y also requires an outlay of $100 now and promises to return $174.90 exactly 4 years hence at time t_4. Both projects have the same degree of certainty. Assuming that the investor's present cost of capital is 10%, which project is more favorable?

Calculations give the following results:

Rate of Return Method			Present Value Method		
Project	Rate	Rank	Project	P.V. Amount	Rank
X	20%	1	X	109.09	2
Y	15%	2	Y	119.46	1

This example has been introduced to show that both methods do not always give equivalent results. By equivalent results, is meant that either technique would:

a. Give the same decision in an accept/reject criterion of an independent proposal.

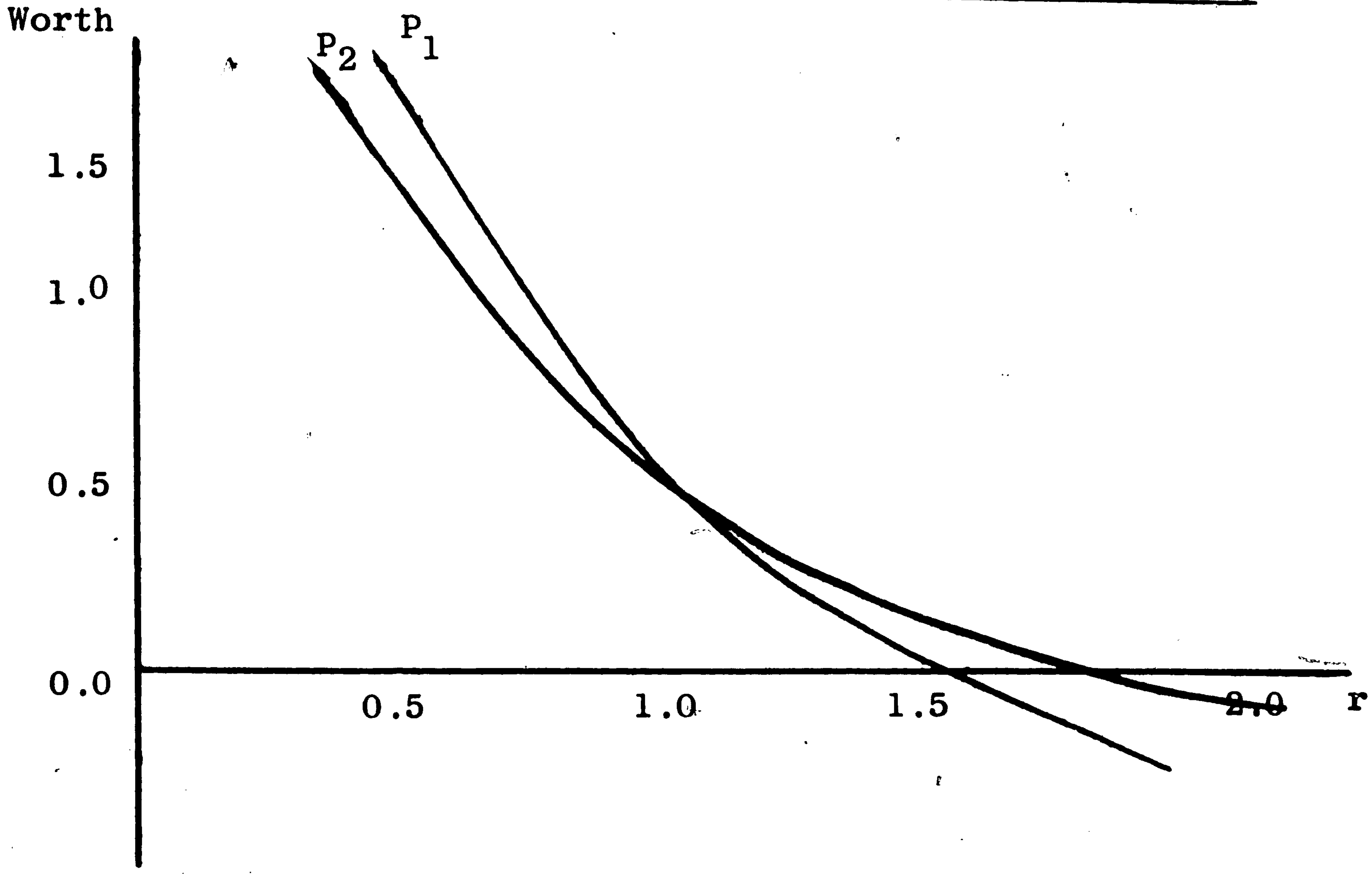
b. Rank order a given number of proposals in the same preference sequence.

When dealing with independent, conventional investments, the internal rate of return will give the same "accept" or "reject" decisions as the present-value method using the cost of capital as the discount rate. "This is so because the net present worth of a project is greater than, equal to, or less than zero if, as, and when the rate of return is greater than, equal to, or less than k (the company's cost of capital)", (30, p. 132). As a general rule, "since most independent investments have cash flow patterns that meet the specifications described above, (i.e., cash flow consisting of one or more periods of cash outlays followed only by periods of cash proceeds), it is fair to say that in practice the yield and present-value methods, would give the same recommendations for independent investments." (4, p. 35)

The problem of correct ranking becomes important when considering mutually exclusive projects. This type of investment is fairly common; e.g., which one of several locations will be best suited for building a new plant. Thus, if a decision maker is considering two mutually exclusive investment projects, P_1 and P_2, it is possible to generate cases such that at a given market rate of interest r, Net Present Worth P_1 > Net Present Worth P_2 while Internal Rate of Return P_1 < Internal Rate of Return P_2. Specifically, if for P_1, the net incremental returns of $E_o = -1$, $E_1 = 1$ and $E_2 = 4$, and for P_2, the net incremental returns[1] of $E_o' = 1$, $E_1' = 2$ and $E_2' = 2$, the internal rate of return for $P_1 = 1.562$ while that for $P_2 = 1.732$. It thus appears that P_2 is better than P_1. But, if the market rate of interest for P_1 and P_2 is $r = 0.50$, then the present worth of $P_1 = 1.444$, while the present worth of $P_2 = 1.222$. Thus, the present worth method indicates that P_1 is superior to P_2. A plot of the present worth of P_1 and P_2 at various interest rates, r, appears as follows in Figure 5.

FIGURE 5

Effect of Rate of Interest on Proposal Ranking



For all possible rates, r, below 1.0, P_1 has a higher present worth than that for P_2, but the internal rates of return give an opposite ranking of P_1 and P_2. Such discrepancies, first analyzed by Lorie

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1 These values are similar to those proposed by Prof. Richard Bernhard (44).

and Savage (66) are based upon certain assumptions for each method:

a. The Internal rate of return method assumes that the rate of interest applied to the investment is also the rate that is to be applied to the reinvested income streams.

b. The net present value method assumes reinvestment at the minimum rate of interest which is the company's present cost of capital.

Thus, "the apparent conflict between the two approaches (methods) results only from differing assumptions that each makes about the future", (78, p. 76). In the example just given, the internal rate of return assumption implies that the net incremental return, E_1 or E_1', at the end of period 1 would be reinvested during period 2 at the same rate of return which the project itself is earning; i.e., 1.562 for P_1 and 1.732 for P_2. The present value method implies that, since the cost of interest was $r = 0.50$, the reinvestment would occur at this rate. The "logic" of this has generated many different arguments and counter-arguments. Ezra Solomon has suggested that the difficulty in using the usual internal rates of return as a ranking device may be overcome by making explicit and consistent assumptions as to the interest rates at which intermediate receipts from projects may be reinvested either in other projects or in the outside market:

"a. The ultimate criterion is the total wealth that the investor can expect from each alternative by the terminal date of the longer lived project.

"b. If the rate of return is to be used as an index of relative profitability, then the relevant rate is the per annum yield promised by each alternative course of action from inception to a common terminal date in the future (usually the terminal date of the longer-lived project).

"c. If the present value is to be used as an index of relative profitability, the expected reinvestment rate or set of rates should be used as the discounting factor. These rates will be equal to the company's present cost of capital only by coincidence. When comparing two projects requiring different outlays, it is necessary to compare present value per dollar of outlay rather than the absolute present value of the projects." (78, p. 77)

Thus, "if a common assumption is adopted, both approaches will always rank projects identically". (78, p. 76) Prof. Ed Renshaw, reviewing Ezra Solomon's comments, claims, "Solomon's over-all rate of return should be interpreted as an average rate of return on funds investment to the common terminal date. The idea of discounting by reinvestment rates is appealing since in committing investment funds, one cannot afford to ignore reinvestment opportunities. The problem of accepting one philosophy of discounting as opposed to the other, of course, exists only if the firm has some grounds for distinguishing between the current cost of capital and future reinvestment rates". (73, p. 81)

Grant and Ireson claim that the average rate of return method is "a misuse of an auxiliary interest in computing prospective rates of return" as well as "a misleading analysis of two investment proposals",[1] (13, p. 505). However, it is easy to show mathematically, that two mutually exclusive proposals requiring the same initial investment,

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1 Another famous argument involving different interpretations about internal rates centers about Keynes' Marginal Efficiency of Capital and Fisher's Rate of Return, with Keynes and Robinson on one side and Fisher and Alchian on the other (15), (40), (76).

$-E_o$, and lasting for the same number of n periods, will be ranked, in the same fashion by Solomon's average rate of return or the present value method:

The present value formula is:

$$I = E_o + \frac{E_1}{(1+r_1)} + \frac{E_2}{(1+r_1)(1+r_2)} + \text{--}$$

$$+ \frac{E_n}{(1+r_1)(1+r_2)\ldots(1+r_n)}$$

Hence:

$$(I-E_o)(1+r_1)(1+r_2)\text{---}(1+r_n)$$

$$= E_1(1+r_2)(1+r_3)\text{--}(1+r_n)$$

$$+ E_2(1+r_3)(1+r_4)\text{--}(1+r_n)+\text{--}+E_n$$

$$=-E_o(1+r^*)^n$$

where r^* is the average rate of return

Therefore:

$$(1+r_1)(1+r_2)\text{--}(1+r_n)I =$$

$$E_o\left[(1+r_1)(1+r_2)\text{--}(1+r_n)-(1+r^*)^n\right]$$

Let

$$C = \text{Constant} = E_o\left[(1+r_1)(1+r_2)\text{---}(1+r_n)\right]$$

Then

$$\frac{C}{E_o} I = C - E_o\ (1+r^*)^n$$

If we have two projects P_1 with present value = I_1 and P_2 with present value = I_2 and r_1^* = average rate of return for P_1 and r_2^* = average rate of return for P_2, and both projects have the same initial outlay and same time periods:

$$\frac{C}{E_o} I_1 = C - E_o(1+r_1^*)^n$$

$$\frac{C}{E_o} I_2 = C - E_o(1+r_2^*)^n$$

Therefore, if $I_1 > I_2$

$$\frac{C}{E_o} I_1 - C > \frac{C}{E_o} I_2 - C$$

and $-E_o(1+r_1{}^*)^n > -E_o(1+r_2{}^*)^n$

But since E_o is negative:

$$1+r_1{}^* > 1+r_2{}^*$$

or $\quad r_1{}^* > r_2{}^* \qquad$ Q.E.D.

Although Ezra Solomon warns the decision maker about employing the present value method on projects requiring different outlays, he did not include such a warning when the internal rate of return method is used. The following example will serve to demonstrate that it is important to consider the size of mutually exclusive investments when using the internal rate of return method:

Investment	E_0	E_1	Rate of Return	Cost of Capital
P_1	-10,000	12,000	20%	10%
P_2	-15,000	17,000	18%	10%

Judging by the internal rate of return, the decision maker would say that P_1 was more desirable. But, this is not a correct decision. As Profs. Bierman and Smidt word it, "A yield of 1,000 % on an investment of a dime is a poor substitute for a yield of 15% on $1,000", (4, p. 37). Actually the important difference between P_1 and P_2 is the incremental benefits derived from the additional expenditure of P_2; i.e., the additional $5,000 outlay for P_2 returns at $5,700. This is actually a return rate of 14% which is certainly more desirable than any yield possible at the market cost of 10%. The above, according to many writers, is a strong disadvantage in the use of the internal return rate method, because "it is necessary to compute the yield on the incremental cash proceeds in order to determine which of a pair of mutually exclusive investments is preferable", (4, p. 40). If more than two mutually exclusive alternatives are being considered, it means that the return on the incremental cash flow of the first two alternatives must be calculated and then the better of this pair decided. The "winner" of the first round is then paired with the third alternative and its incremental benefit return is calculated; etc.

Because of the discussion in this section and the possibility of multiple return rates when the discounted cash flow method is employed, many writers prefer the present value method to that of the discounted cash flow technique. M. Klein states, "The net present worth method is a better measuring and selecting tool than the rate method although the latter is usually preferred by managers and engineers", (64, p. 64). Prof. Bernhard writes, "We have earlier concluded that the present worth method is superior to the internal rate of return method for ranking productive investments proposals", (44, p. 26). Profs. Bierman and Smidt claim, "When the two methods lead to different decisions, the present value method tends to give better decisions. ...For most of us, the present value method is simpler, safer, easier and more direct", (4, pp. 34, 46). Some writers see little or no difference between both methods. For instance, Joel Dean and Winfield Smith write, "These net cash flows are then discounted to yield either a measure of rate of return or of present value, which then becomes the criterion for accepting or rejecting the proposal and for comparing it with other proposals", (48). Even Grant and Ireson claim that, "Once a particular minimum attractive rate of return is selected as the criterion for comparing alternatives, a correct analysis of rates of return will invariably lead to the same conclusion that will be obtained from a correct annual cost comparison or a correct present worth comparison", (13, p. 121).

Some writers avoid bringing the present value method into consideration and merely deal with the discounted cash flow technique. These writers generally act as if they have found the answer to the investor's dream with this one method. Examples of this approach are Joel Dean's (47), John G. McLean's (69),[1] Edward A. Ravencroff's (72),

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[1] This article is the one included in the finance section of Bursk's and Chapman's, New Decision Tools for Managers, Harvard University Press, 1963.

Ray I. Reul's (74) articles.

Of course, many investigators[1] are dissatisfied with either method, because they lack a proper modification for uncertainty, but for an operational, day-to-day technique, these two methods appear practical. Actually, "if a corporation knows its cost of capital (at least approximately) and can either obtain additional funds from the market at that cost of capital, if desirable internal investments are available, or can invest any excess funds externally at the cost of capital, then either of the two discounted cash flow procedures can be used to make correct investment decisions", (4, p. 43).

3.4 Subjective Method

3.41 General

Strangely enough, in articles or textbooks dealing with capital investment decision making, it is rare to find the subjective method included in any list of operational investment decision procedures or measures. Yet, this method is widely used by businessmen of big and small firms. Prof. Istvan verified this when he surveyed the investment "habits" of 48 major companies in the United States.[2] He found that four firms employed subjective judgement for all proposals, and that all of the remaining 44 firms employed this technique in various

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[1] For example, R. M. Adelson's comment, "Although the present worth method appears to be gradually gaining acceptance, the present worth approach and the way in which it is generally applied still leave some rather growing doubts", (39, p. 21).

[2] Refer to (14).

degrees.[1]

The subjective judgement criterion operates in the following types of environment:

a. In situations when no other measure is really applicable. An example of such an occasion might be a proposal to expand the company's cafeteria facilities for the comfort of employees during the lunch hour. In this case, no operating advantages can be determined, and only subjective judgement can decide the acceptability of the expenditure. But, Prof. Istvan warns, "that where the operating advantage can be determined at all, subjective judgement should not be the sole measure of acceptability", (14, p. 94).

b. In specific cases of necessity when the proposal in question is known to have an inferior return capability. Such a situation could involve relatively poor investments made for the sake of maintaining a competitive position by the firm. Actually, this class of subjective judgement criterion is equivalent to the necessity/postponability criterion described in 3.53. In view of this, it can be stated that, when projects of necessity are being studied and evaluated, all firms use subjective judgement.

c. In investment cases where objective data is replaced in part or in full by subjective judgement. Such circumstances

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1 "All of these companies are among the ten largest in their respective industries and more than two-thirds are among the six largest." (14, Preface).

usually arise when, due to the urgency of a proposed expenditure, there is little or no time for data collection and evaluation. Opinions, hunches, experience, and rules of thumb, come strongly into play to decide the fate of such a proposal. In these cases, "subjective judgement is a poor measure of acceptability. It does not measure anything concretely. It does not account for time-pattern effects. It does not provide for objective comparison of proposals nor does it set a predetermined minimum", (14, p. 94). This is especially true for those companies which use the subjective judgement criterion for evaluating all proposals. But, why are they still successful? For example, one of the leading airline companies that makes upwards of $20 million of expenditures annually uses subjective judgement exclusively. Its vice-president of finance stated that once it is determined how many plans will be needed (based on a market forecast of passenger miles), there is no need for evaluation of further expenditures because all expenditures are based on providing facilities to service repair, and maintain the aircraft.[1]

3.4 Subjective Method

3.42 Angell's Hypothesis

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1 Refer to (14, p. 95). It should be noted that the major competitor of this airline company measures all expenditures by an average simple rate of return, after complete analysis.

Economics Prof. James W. Angell of Columbia University has investigated and hypothesized on the way the successful decision maker subjectively manipulated his "data" and judgements to arrive at a "sound" decision in the face of uncertainty. Prof. Angell has developed his hypothesis as an alternative to the probability approach and to Shackle's Proposal. He has attempted to put into a formula what he believes expresses the decision maker's "thinking":

Let y = Most likely gross gain outcome which includes both recovery of sum originally invested and any net profit, expressed as a per cent of the current market price, of the investment. $y \geq 1\%$.

x = Coefficient of maximum likelihood of receiving at least y. $0 \leq x \leq 1$.

t = Time period required to receive the gain in full.

a = Coefficient (perhaps exponential) embodying the appropriate allowance for the time factor t.

Φ = Measure of gross satisfaction expressed as satisfaction per dollar to be invested since y is expressed as a per cent of the cost of investment.

y' = Most likely loss, expressed as a per cent of the cost of the investment.

x' = Coefficient of maximum likelihood of y'.

Φ' = Measure of pain or dissatisfaction, per dollar invested.

ψ = Net satisfactory per dollar invested.

Then he hypothesizes that:

(1) $\Phi = \Phi\left(\frac{xy}{at}\right)$

(2) $\Phi' = \Phi'(x'y')$

(3) $\psi = \Phi - \Phi' = \psi\left(\frac{xy}{at} - x'y'\right)$

The maximum-likelihood value of the most likely loss, x'y', cannot be discounted to present-value equivalents, nor can the time factor be introduced into it in any normal manner. The reasoning for this rests upon the assumption that there is no way of estimating when the loss, if there is one, will be realized. This has the effect of weighing a loss heavier than an equivalent gain.

If the value of ψ is positive, the investor then compares this value with the desirability to him of continuing to hold part or all of his money. Then, if:

U_M = desirability of retaining all the initial invested money M.

I = amount of money which would be spent for the project.

and $\frac{d\psi}{dI} \geq \frac{dU_M}{dI}$, the decision maker will adopt the proposal. Although, neither the net satisfaction nor desirability of holding onto uninvested money "can be expressed by the investor in consciously quantitative terms, it is clear that comparisons are made in the actual world and that decisions are reached in terms of 'greater or less' if not in terms of cardinal magnitudes", (41, p. 20).

"The decision-making process through which investors go, if the foregoing hypothesis is at all correct, is thus a complex affair. The major elements in this process are largely subjective; they can usually be expressed in quantitative terms only rather loosely and inaccurately, if at all. The relations among them are not stable over time, and the investor's own appraisals of them change frequently. Hence, prediction of any one investor's decisions is probably impossible." (41, p. 27)

Thus, Prof. Angell's hypothesis has been one attempt to formalize what may be the thinking of the successful businessman. To date, encouragement for his hypothesis has come from the composite results of groups of decision makers rather than from individual decision makers.[1] Realistically, Prof. Istvan believes that the businessman who uses subjective judgement merely employs some self-developed payback or rate-of-return yardstick.

3.4 Subjective Method

3.43 Shackle's Proposal -- It has been included for two reasons:

a. It is widely known in Europe and less familiar in the United States.

b. It attempts an approach different from the calculus of probability in its attack upon uncertainty.

Prof. Shackle's original work entitled, "Expectations in Economics", was published in 1949. His most forceful argument against the use of probability coefficients as indices for investment outcomes rests on the fact that probability calculus uses a form of knowledge which is derived from the outcome of a <u>long</u> series of <u>identical</u> past trials. As such, prediction of the outcome of any one future trial cannot be made "but only of the average outcomes of a long series of future trials, all identical to those conducted in the past", (41, p. 4). Investment acts are "one shot", unique affairs which Prof. Shackle argues, do not meet

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1 We may be entering the realm of descriptive analysis which rightfully belongs to the psychologists. Much has been written on this process. A recent (1965) article, entitled, Human Decision Making Under Uncertainty and Risk: Computer Based Experiments and a Heuristic Simulation Program, shows how the computer may be used to assist the psychologist in this endeavor (52).

any of the prerequisites demanded by the rules of probability.[1]

Prof. Shackle's hypothesis introduces the concepts of potential surprise and degrees of enjoyment. Potential surprise[2] is the amount of surprise the investor would expect to feel if a particular outcome were actually realized. This surprise is generally the inverse of the investor's estimate of the likelihood of that outcome's occurrence. In general, "the intensity of enjoyment of a given hypothetical outcome by imagining it in advance is no doubt a function of several variables but two of these are likely to be dominant; this intensity will plainly be an increasing function of the desirability of the outcome in question, and a decreasing function of the degree of potential surprise associated with it", (28, p. 18). This statement is expressed as an equation called the stimulation function[3]: $\Phi = \Phi\,[x, y(x)]$, where x is the anticipated value and y(x) is the associated potential surprise. The two values of x which maximize Φ are given special names, "primary focus gain" and

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1 Prof. Shackle lists these prerequisites in Chapter I, p. 6 of his book and uses Chapter VII to argue against the use of probability calculus in investment decision making (28).

2 Prof. Shackle speaks about potential surprise as follows: "Thus we shall say that a person can compare his own respective degrees of belief in two different outcomes of some course of action or two different answers to a question by taking each of these outcomes or answers in turn and asking himself what intensity of shock or surprise he would feel if without there having been meantime any change in the knowledge available to him on which he based his belief in it, he were to learn that this belief is mistaken. The measure so obtained is what we may call the potential surprise associated, by a particular person at a particular date, with the falsity of the answer or the non-occurrence of the outcome", (28, p. 11).

3 It can also indicate "distress by anticipation" "if the outcome in question, instead of being desirable, is positively hurtful or disadvantageous", (28, p. 18).

"primary focus loss"; they are related to the "greatest degree of enjoyment" and the "greatest degree of hurtfulness" to the investor.

In making a decision, the investor selects that investment or group of investments which offers to him the optimum combination of gains, possible losses, and the associated degrees of potential surprise, based upon his own "gambler-preference" map.[1] This "map" is defined as follows: "For any specific focus-loss there will be some specific focus-gain such that if he is faced with this pair of focus-outcomes his situation seems to him neither more nor less desirable than if he had the assurance of experiencing neither gain nor loss. We shall say that this focus gain compensates the focus-loss. The ratio of focus-loss to its compensating focus-gain will in general be different when the focus-loss is different. The set of all such ratios obtained by varying the focus-loss, other circumstances remaining unchanged, is what we mean by the schedule of gambler preference of the given individual in these circumstances." (28, p. 33) These focus outcomes are in effect simply different values of x, since Prof. Shackle states, "Before any further step (i.e., after primary focus gain and loss are determined), can be taken in comparing the merits of two rival ventures, for each of which the primary focus outcome have been determined, these primary focus-outcomes must all be replaced by equivalents carrying nil surprise. Each of these equivalents will be a value of x, say x_s such that if the primary focus outcome is

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[1] In addition to this "map", Prof. Shackle develops the concepts of gambler's indifference map, and gambler's opportunity curve.

(x_1, y_1) then $\Phi(x_s, 0) = \Phi(x_1, y_1)$", (28, p. 25).

Prof. Angell of Columbia University, reviewing Shackle's Proposal thus far, believes that these propositions, taken on their face, seem entirely reasonable, but other notions, which Shackle also advances, seem to him more debatable. These other notions are:

a. An investor may attach the quality of zero potential surprise both to a particular gain outcome or range of gain outcomes of a particular investment and to a particular loss outcome or range of outcomes. This concept to Prof. Angell seems unreasonable because, "If a particular gain outcome from a particular investment would convey zero surprise, were it to be realized, surely no one loss outcome from that investment would also convey zero surprise (unless, perhaps, when both outcomes are close to zero). The investor is unlikely to regard both a particular gain and a particular loss as outcomes of zero surprisingness, (or of maximum unsurprisingness!)" (41, p. 5)

b. The use of "focus" gain and loss, i.e., the prospective gain and loss outcomes, for comparing investment. In effect, Prof. Shackle tries to describe the philosophy of decision-making by the investor. The investor primarily decides, from among the various possible gain outcomes, which one, if realized would usually entail some degree of potential surprise, but which also offers a somewhat larger prospect of gain than do those gain outcomes for which the attached potential surprise is less or zero, thus insuring the greatest enjoyment. Actually, the investor is trading off more prospective gain for less potential surprise until he reaches a prospective gain outcome offering a combination of prospective gain and attached potential surprise from which he believes he would anticipate more enjoyment than from any other combination. This holds conversely for his calculations of possible losses. These then become the prospective gain and loss outcomes; i.e., his focus gain and loss, which he uses in making comparisons with other investments. Prof. Angell believes that an investor does not act in this fashion, but, rather, employs a most likely gain outcome and the most likely loss outcome. He then "decides whether the corresponding prospective gains and losses, when viewed in the light of the associated likelihoods (in effect, gain or loss times its likelihood), meet the requirements of his own preference system. Once his estimates of the most likely gain and loss outcomes are made, all other possible outcomes cease to interest him." (41, p. 6)

c. The question of zero potential surprise. Actually, the potential surprise associated even with the most likely outcome is greater than zero. "It takes an extraordinary smug investor to expect no surprise at all if his guess turns out to be right, even if it is his very best guess!" (41, p. 6)

d. The continuity of the stimulation function, Φ, which Prof. Shackle merely assumes. If what Prof. Angell states is the more correct mode of decision making, it may be that few of the possible values are really ever consciously present in the investor's mind.[1]

e. The lack of a direct method for detailing how the investor weighs a prospective focus gain and its associated potential surprise against the related prospective focus loss and its surprise, in order to arrive at an estimate of the net enjoyment or distress he could anticipate were he to make the investment.

Perhaps, it would be best to summarize this section with the following quote from Prof. Shackle's book:

> "But my system is no whit more indeterminate than theirs (i.e., those who favor the use of the probability calculus). Both they and I reduce the enterpriser's conception of the gain and loss possibilities of a venture to a pair of numbers, precise numerical magnitudes, one of which represents and summarizes the favorable and the other the unfavorable potentialities of the ventures, as judged by the enterpriser. The difference between the characters of the two resulting expressions is, first, that their figures for "yield" and "risk" are more mathematical abstraction, the answers to a particular kind of sum, while mine are claimed to represent psychological realities, the levels of hypothetical outcome which generate the highest intensities of feeling; and secondly, that their figures and mine are, of course,

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1 An excellent discussion of this topic is found in Symposium on Uncertainty and Business Decision (eds. C. F. Carter, G. P. Meredith, G. L. S. Shackle), Liverpool, 1954, pp. 40, 50.

determined in two entirely different ways." (28, p. 124)

3.5 Other Techniques

For the sake of completeness, the following investment decision techniques are included. No complete review and analysis of any method will be undertaken. Instead, each method will be listed together with a brief "write-up".

3.51 M A P I[1] Formula

The MAPI formula was developed as a means of facilitating the expenditure evaluation calculations when hundreds or even thousands of replacement proposals are reviewed annually by a firm. "It was conceived and presented as a replacement formula (the term replacement being broadly construed to include mixed replacement-improvement-expansion situations), its main purpose being to indicate the proper timing of re-equipment decisions." (33, p. 14) The end result of the MAPI formula is a figure called the "next year rate-of-return". It is the rate of return that will be obtained by making the concerned investment immediately rather than putting it off for another year. It is derived as follows:

a. Operating Advantage (of proposed new facility for next year, before taxes and capital consumption).

Plus

b. Avoided capital expenditures by making replacement (for next year only).

Less

c. Income taxes (on sum of (a) and (b))

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1 MAPI (Machinery and Allied Products Institute) formula was developed by George Terborgh, Research Director, Machinery and Allied Products Institute. The basic reference to MAPI is (33).

Less

d. Capital consumption costs (derived from chart).

Divided by

e. Net investment to be made in new facility (after salvage and the next year avoided capital expenditure).

Equals

f. Next-year rate of return on proposal.

Business Week (September 27, 1958) describes the charts (step "d" above as follows:

> "The charts themselves are produced from an elaborate mixture of: the rate at which the new project's earnings will decline; the service life of the new equipment, its final value for sale, trade-in or scrap; the corporate tax rate, the company's depreciation system; the ratio of the company's debt to its total investment, the interest rate it pays on borrowed capital, the after-tax return it gets on equity capital.
>
> "The MAPI charts have precomputed all these things. They have assumed that a 25% debt ratio, a 3% interest rate, and a 10% after tax return on capital are about the normal for most companies. There are three charts--because MAPI offers the analyst three general patterns in which the earnings of a new machine will decline over the years of its service life. In some cases, the machines will produce half their earnings by the time they have worked half their service life; in others, they'll run off one-third of their earnings when their service life is half over; some will produce two-thirds of their earnings in half their estimated service life."

According to Prof. Istvan, "When the actual conditions in a firm conform to those assumed in the MAPI formula, it is an excellent measure of acceptability. It accounts for all factors relevant to the problem including the time pattern of capital consumption. Whether its assumptions are realistic, however, remains a question....It is difficult to accept a 25% debt ratio, a 3% interest cost and a 10% return of equity---There is no reason, however, why the formula would not be adapted to reflect the capital mix and costs of the individual firm", (14, p. 75).

Ray I. Reul worded his very strong sentiment against MAPI as follows: "This approach (MAPI) is an interesting intellectual flight of the imagination. Many of the assumptions necessary to make its application practical are completely unrealistic. The basic assumption is that history will repeat itself endlessly and that our need for the facility for which replacement is being studied will go on forever. It assumes uniform, regular changes in variables that are not at all realistic, either.... The answers are relative--the comparative values changing in an irregular fashion with changes in the interest rate assumed." (74, p. 131)

Strangely enough, when MAPI was first introduced, many firms attempted to explore the feasibility of a unified capital-controls program covering the firm's investment expenditures for all purposes based on the MAPI concept. Accordingly, there was a requirement for an investigation of the way in which the MAPI method measures the productivity of capital as compared with other methods. Joel Dean and Winfield Smith undertook the job[1] to prove that MAPI cannot be used for such a unified method and that it compares very unfavorably with the rate of return and present worth methods in handling all but replacement investment decision making. George Terborgh replied to Dean and Smith in (80) and literally tells them "I told you so". According to Terborgh, "No knowledgeable person has ever claimed universality of application of the MAPI formula.... It was offered as an improvement over the primative rules of thumb so widely employed in

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1 Refer to the well written article (48).

American industries to make these (replacement) decisions. As such, it has proved very useful, as evidenced by the extent of its employment", (80, p. 307). But, the last part of this statement may be in doubt today because:

a. Prof. Istvan's study in 1961 of 48 leading firms showed that only two companies employed the MAPI formula for measuring acceptability of their replacement expenditures; but, "neither employs it in the exact manner developed by Terborgh", (14, p. 75). These firms "built in financing assumptions of their own, stating that the assumptions employed by Terborgh do not compare closely enough with the actual situation. One of the firms done away with the chart altogether ...", (14, p. 75). Of the firms not using MAPI, executives of 16 companies stated that they had never heard of MAPI and the executives of 26 companies stated that they were familiar with the MAPI formula, had considered it for use in their firms, but had decided against adopting it.

b. Recent articles on equipment investment decision making do not even mention MAPI.

Perhaps in fairness to MAPI, "it could probably be shown that this measure is more widely used in smaller firms, where the expenditure and evaluation environment are uniform throughout the firm", (14, p. 77).

3.5 Other Techniques

3.52 Accounting Methods

Basically, any calculations used to generate a ratio relating earnings to investment which yields a simple rate of return has been designated as an accounting method. The big problem with this method is its many variations. Some businessmen use an initial investment concept while others employ an average investment concept or book values, and still others fail to account properly for salvage value. Also, some earnings are gross or net of depreciation, either before or after taxes, and some are the average of several years or the first year only. Naturally, such a wide diversification gives different results. "This shortcoming can be minimized only by arbitrarily standardizing on one variant of the method and making all computations according to this standard."[1] (47, p. 128)

The usual variations of the accounting method are:

3.521 Initial Year Simple Rate of Return

This rate of return is computed as follows:

$$r = \frac{\text{Initial-year net operating advantage}}{\text{Initial Investment}} \times 100$$

"The net operating advantage should be determined after financing costs, capital consumption allowance, and income taxes. Investment should be net of salvage on any replaced facility but before future salvage on the proposed facility." (14, p. 78) Prof. Istvan in (14),

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[1] John G. McLean describes in (69), his "struggle" to unify the different rate of return methods used by the various divisions of his company.

notes that many businessmen erroneously believe that it is not necessary to account for financing costs, capital consumption and income taxes because all proposals will be subject to these factors and hence it is possible to eliminate them without altering the percentage relationship between different proposals.

It is important to note that the initial year net operating advantage will depend on the adopted method of tax depreciation, type and cost of funds used, and relative lives of the investment proposals.

Under certain circumstances, this measure may be used as an index for the acceptance of a proposal, i.e., "when it is, for all proposals being considered, indicative of the average of the yearly rates of return. If this condition is not met, misallocation of funds may result in addition to that misallocation resulting from neglect of the time dimension of future advantages", (14, p. 81).

3.522 Average Simple Rate of Return

There are actually two different procedures to calculate this rate of return. One of these is:

$$r_1 = \frac{\text{sum of the yearly simple rates of return}}{\text{number of years of expected proposal life}}$$

In calculating this return, the same factors used in calculating the initial year rate of return must be taken into account, except that financing costs, capital consumption and income taxes are considered for each particular year. Also, the investment for each year is the investment of the preceding year minus the capital consumption of the preceding year. The last year's investment figure should be net of salvage.

The second method, although it gives the same result as the first, employs a different method of calculation:

$$r_2 = \frac{\text{average net operating advantage}}{\text{average investment}} \times 100$$

"The average simple rate of return can serve fairly well as a measure of acceptability. The percentage figure obtained from the calculation is a fair indication of profitability." (14, p. 82) The reason why Prof. Istvan uses "fair" lies in the fact that it fails to take into account the time shape of the yearly net operating advantages and yearly investment figures.

In this method, the type of tax depreciation pattern used, does not affect the average net operating advantage. The elimination of taxes from the calculation does not affect the ranking of proposals but it does affect the indications for profitability.

3.523 Book Return on Book Investments

The rate of return for this technique is usually defined as the average income after depreciation divided by the average book value of the investment times 100:[1]

$$r = \frac{\text{(average income-average depreciation) for life of proposal}}{\text{average book value of investment}} \times 100$$

In applying this formula, the pattern of depreciation affects both

[1] Some businessmen use a ratio which has the same numerator but change the denominator to original cost of investment without consideration given to accumulated depreciation. According to Profs. Bierman and Smidt, "The use of undepreciated cost has certain advantages over the use of book value. These advantages are not so important in capital budgeting and are relatively unimportant compared to the failure to take into consideration the timing of the cash proceeds", (4, p. 22).

the numerator and the denominator, and usually gives different results.

"The income on book value (may be) a common and useful measure of performance, but it is less useful as a device for ranking investments." (4, p. 21). In fact, seldom does this method give the same ranking of investment proposals as does the discounted cash flow procedures.

Rather than look at one figure for the entire life of the proposal, some decision makers employing this method, prefer to "see" the yearly pattern of returns for each proposal. The following examples due to Ray I. Reul will be used to demonstrate how this method can lead to ridiculous results. Assume an investment of $1,000 is required. The net income before depreciation and income taxes is estimated at $400 per year for five years. Depreciation is to be taken at 20% per year by the straight line method. The income tax rate is 50%. In tabular form, the rate of return for each year can be derived as follows:

TABLE III

Straight Line Depreciation and the Book Return on Book Investment

Year	Net Income Before Tax	Depreciation (Straight-Line)	Profit After Tax	Book Value	Rate of[1] Return
1	$ 400	$ 200	$100	$1,000	10.0%
2	400	200	100	800	12.5%
3	400	200	100	600	16.7%
4	400	200	100	400	25.0%
5	400	200	100	200	50.0%
Total	$2,000	$1,000	$500	-	-
Average	400	200	100	600	22.8%

[1] Rate of Return for each year = $\frac{\text{Profit After Tax}}{\text{Book Value at Beginning of Year}} \times 100$

If this project continued to produce an income in the sixth year, then, the rate of return would be infinity.

To study the effect the pattern of depreciation used has on the rates of return, all data will remain the same, except that the sum of the digits method will be used to evaluate yearly depreciation.

TABLE IV

Sum of Digits Depreciation and the Book Return on Book Investment

Year	Net Income Before Tax	Depreciation (Sum of Digits)	Profit After Tax	Value	Rate of[1] Return
1	$ 400	$ 334	33	$1,000	3.3%
2	400	266	66	666	9.9%
3	400	200	100	400	25.0%
4	400	133	134	200	67.0%
5	400	67	167	67	250.0%
Total	$2,000	$1,000	$500	-	
Average	400	200	100	666	71.1%

Obviously, we have obtained a completely new set of figures for the rates of return.

It is also possible to show that if investments, similar to the one just described, were made each year for seven years, the rates of return start to increase rapidly as soon as investments are stopped.[2] These rates are:

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[1] Rate of return for each year = $\frac{\text{Profit After Tax}}{\text{Book Value at Beginning of Year}} \times 100$

[2] In financial circles this is called "milking the franchise", i.e., the amount of profit goes down but the rate of return goes up.

Year	Rate of Return
1	10.0%
2	11.1%
3	12.5%
4	14.3%
5	16.7%
6	16.7%
7	16.7%
8	20.0%
No 9	25.0%
Invest- 10	33.3%
ments 11	50.0%

Because of such examples, Ray I. Reul believes, "the ratio of annual profit to book value is a vague and meaningless figure. Since this ratio is not even consistent with itself, of what use would be a method that produced comparable inconsistent evaluations"? (74, p. 119).

As an overall evaluation of accounting methods, Prof. Istvan states, "It provides a percentage result that is indicative of the profitability of the proposal and that lends itself to comparison with the percentage results of other proposals or with a minimum level of acceptability. It can, therefore, be used in the allocation of funds", (14, p. 77). Prof. Istvan seems to underplay the time pattern effects on the net operating advantage and investment when he made the above statement. To Joel Dean this is the prime factor to disqualify the accounting method for even consideration as a good yardstick: "more serious drawback to the use of the accounting method is that it is insensitive to variations in the time pattern of investment outlays and earnings. By taking an average of earnings over the life of the project, this method ignores the earning trend, which may be quite important. ...Only a company whose investment projects are roughly similar in time shape and in economic life can ignore this feature", (47, p. 128). Ray I. Reul states, "It, (the accounting method), will be in serious error whenever the time pattern of cash flows varies appreciably in the alternatives being compared", (74, p. 131). Ezra Solomon agrees and claims, "Without these conditions, (i.e., when outlays occur at a single point of time, when expected benefits flow evenly over the life of the project, and where economic life corresponds to the life assumed for bookkeeping purposes), the results derived from this approach (i.e., accounting method, especially book return on book investment) are subject to fairly wide errors", (30, p. 125).

3.5 Other Techniques

3.53 Necessity/Postponability

Necessity[1] and postponability, widely used standards for selecting investments, are opposite faces of the same coin.

Postponability operates on the principle that a firm should undertake first those projects that must be undertaken now, if ever. Those that are postponable can be put off to a later date. This is especially true in situations where there is an excess of budget proposals over available funds.

Some writers,[2] fail to acknowledge the fact that there is a place in the capital budgeting machinery for the necessity/postponability

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1 Necessity or degree of necessity is the degree of urgency of a proposal; i.e., the extent to which it cannot be postponed.

2 Joel Dean in (9) gives the impression that there is almost no justification for postponability, but in (47), he definitely claims that there is a logical place for the necessity/postponability criteria.

criterion. Investment proposals which must be made to meet certain government and legal requirements have a high degree of necessity and cannot be postponed. The replacement proposal of a very important piece of equipment; e.g., replacement of the telephone company's micro-wave relay tower destroyed by lightning, could fall into this high degree of necessity classification.

Such projects acquire their high priority from the inherent immense productivity and/or immense company profits which are imbedded in the proposal. But, as a general rule, "must do" projects of this nature "seldom bulk large in the company's over-all capital expenditure program", (47, p. 123).

The basic problem with the necessity/postponability criterion lies in the fact that "it is not logical and is not likely to lead to allocation of investment that produces maximum profit. A large proportion of investments that would yield big savings and high profits could be put off almost indefinitely", (9, p. 20). Many examples can be cited to validate this belief.[1] Accordingly, there does not seem to be any inherent relationship between postponability and profitability.

Another problem with this criterion is the fact that degrees of urgency cannot be measured according to a standard. There is seldom any general agreement as to the priority ranking of urgent or non-postponable projects. This usually generates a contest of

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1 One of the examples used by Joel Dean refers to a plant modernization project which may be highly postponable; but which, if it can produce annual savings which will yield 30% on the added capital tied up, is to be preferred to a less postponable but less profitable project, (47, p. 123).

personalities. The victor being the more eloquent and/or persistent division head.

Thus, by its very nature, postponability can "forestall expansion investment and technological advance", (9, p. 28). It is for these reasons that some writers claim this criterion fosters a stagnant modus operandi.

3.5 Other Techniques

3.54 Decision Trees

Since "the investment problem is not posed in terms of an isolated decision, (because today's decision depends on the one we shall make tomorrow), nor yet in terms of a sequence of decisions, (because under uncertainty, decisions taken in the future will be influenced by what we have learned in the meanwhile)",[1] (18, p. 250) the most recent investigations in the field of capital investments have dealt with decision trees and risk analysis.[2] These tools are needed by the decision maker who cannot analyze a project as if it existed in vacuo without giving ample consideration to any other problems which he faces at present or he may have to face in the future.

Decision tree analysis, a form of dynamic programming, is a convenient method for representing and analyzing a series of investment

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[1] Peter F. Drucker expresses the relationship between present planning and future events as: "Long range planning does not deal with future decisions. It deals with the futurity of present decisions", (49, p. 239).

[2] For examples refer to a 1962 textbook, (18, chapt. 6); three 1964 articles (68), (67), (57); a 1965 article (58); and the 1965 textbook, (22, chapts. 1-6).

decisions to be made over time. The essential characteristics of all decision tree analysis involves the following:

1. A choice or in some cases a sequence of choices must be made among various possible courses of action.
2. This choice or sequence of choices will ultimately lead to some consequence, but the decision maker cannot be sure in advance what this consequence will be because it depends not only on his choice or sequence of choices but on an unpredictable event or sequence of events.
3. There exists, either by statistical research techniques or personal judgement or any combination of these, a probabilistic quantification to designate how good the chances of achieving each consequence are believed to be.
4. There also exists a quantification of preference by the decision maker to describe how good the chances or odds of obtaining any one consequence (C_1) would have to be to make him willing to gamble on either of any other two consequences (C_2 or C_3) rather than to take C_1 for certain.

Operational investment writings which appear in periodicals employ the first three factors only,[1] while recent textbooks[2] on statistical decision making, use all four factors for a complete theoretical analysis. The fourth factor involves the concepts of utility, lotteries,

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[1] Refer to (58), (68), (67).

[2] Refer to (26), and (22).

and conditional probabilities, which are covered, in part, in Chapter V of this thesis.

Although these periodical articles do not include the fourth factor, the fact that the decision maker(s) probably do consciously or subconsciously employ some mental manipulations which involve utilities to arrive at a final decision can be implied.

The following example[1] will serve to illustrate the above factors:

A firm is faced with the decision of introducing a new product regionally or nationally with the related capital investment problems. All the pertinent decision points, chance event nodes and probabilities involved can be diagramed as follows:

[1] Related to problem employed by Hespos and Strassman (58).

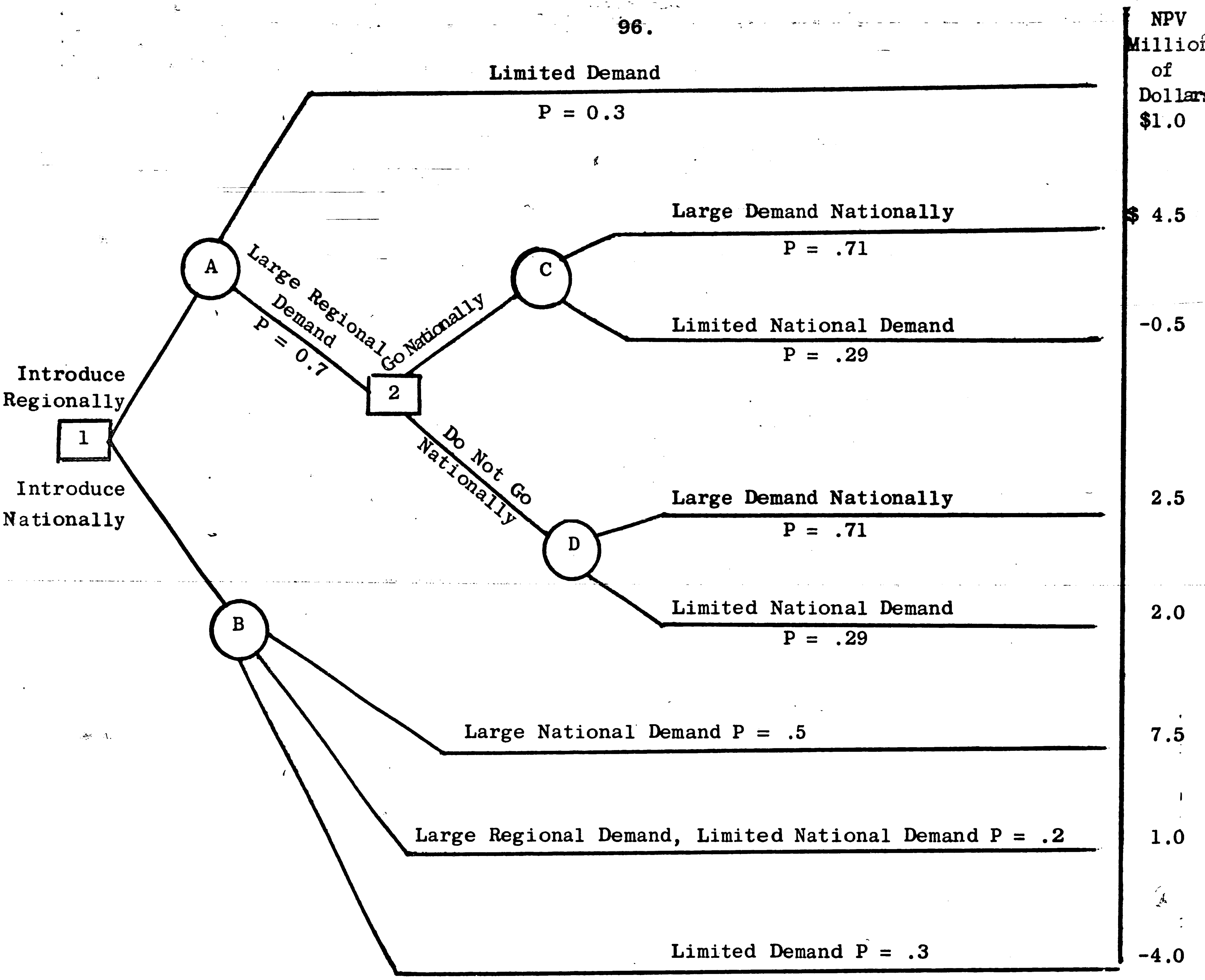

Figure 6

Example of Standard Decision Tree Method

NPV = Net Present Value (accounts for time differences).

= Decision Point

= Chance Event Node

P = Probability (All probabilities from any branch must sum to 1.0).

An analysis of this diagram to determine an optimal sequence of decisions involves starting at the final consequences and "rolling backward". At each chance event node, the expected net present value is calculated for all of its outgoing branches. At each decision point, the expected net present value is calculated for all of its outgoing branches. At each decision point, the expected net present value is calculated for each of its emanating branches and the highest is selected. In either case, the expected net present value of that node or decision point is carried back to the next chance event node or decision point by multiplying it by the probabilities associated with the branches that it travels over. Thus, referring to Figure 6, the expected net present value of all branches emanating from chance event node C is \$3.05 million (\$4.5 x .71 + (- \$0.5) x .29). In like fashion, the expected net present value at node D is \$2.36 million. Rolling back to decision point 2, the alternative with the highest net present value; i.e., "distribute nationally", with NPV of \$3.05 million, is selected by the decision maker. Continuing the roll back technique, it is seen that the branches emanating from chance event node A has an overall expected net present value of \$2.44 million (\$1 x 0.3 + \$3.05 x 0.7). Similarly, the expected present value at node B is \$2.75 million. Hence, the alternative that maximizes expected net present value of the entire decision present value of the entire decision tree is "introduce nationally" at decision point 1. The procedures and thinking engaged in, to arrive at an optimum "path" is closely related to the selection of an optimum strategy by a player in the theory of game.[1]

[1] Refer to Appendix 3, Game Theory - A Brief Review

As can be verified by the above example, the use of decision trees is based on three assumptions:

a. All the states of the world; i.e., alternatives and chance events, that could occur in the future can be enumerated and defined in advance.

b. It is possible for the decision maker (or an associated group), to assign probabilities and costs to each state that may occur in the future.

c. The optimum for any course of action is obtained by maximizing or minimizing mathematical expectations; i.e., maximization of profits or the minimization of costs for equivalent services.

To validate these assumptions as much as possible, it is necessary to obtain "accurate" cost estimates, business forecasts, probabilities, etc. To this end, John F. Magee believes, "Marketing analysis, operations research, engineering analysis, and financial analysis have vital roles in investment analysis. ...Those engaged in the analysis should be encouraged to express doubts, and uncertainties and to express estimates of costs, technical feasibility, or forecasts of market conditions in terms of ranges or probabilities", (67, p. 80). But, how does one incorporate such factors of uncertainty and doubt into decision trees analysis? At present, there appears to be two approaches:

a. John F. Magee's[1] recommendation for the use of three

[1] Prof. J. Morley English reviews J. F. Magee's decision tree technique in Engineering Economist, Vol. 10, No. 1, Fall, 1964.

estimates; i.e., the optimistic estimate, most likely estimate, and pessimistic estimate for each doubtful factor; e.g., demand forecast. Incorporate these estimates as separate branches emanating from a chance node. All other procedures and criteria remain the same.

b. R. F. Hespos' and P. A. Strassman's method of stochastic decision trees. This most recent of methods is similar to the conventional decision tree approach, except that it also includes the following features:

1. All quantities and factors, including chance events, are represented by continuous, empirical probability distributions. In effect, this replaces the single best estimate of the standard method.
2. The information about the results from any or all possible combinations of decisions made at sequential points in time can be obtained in a probability form.
3. The probability distribution of possible results from any particular combination of decision can be analyzed using the concepts of utility and risk.

The stochastic decision tree analysis involves computer simulations, and according to the orginators, seems ideally suited to the computer language known as General Purpose Systems Simulator (GPSS). On each iteration in the simulation, a value of each factor is randomly selected from the appropriate frequency distribution and used in the computation. It is now also possible, by use of the computer, to evaluate by complete enumeration all of the possible paths through the trees, regardless of the number. (By the standard method, simplification of a very large number of paths can be accomplished on the basis of dominance and the setting of an arbitrary standard which cannot be exceeded if the path is to be considered.)

A simplified diagram of the same example used earlier, employing stochastic decision trees would appear as:

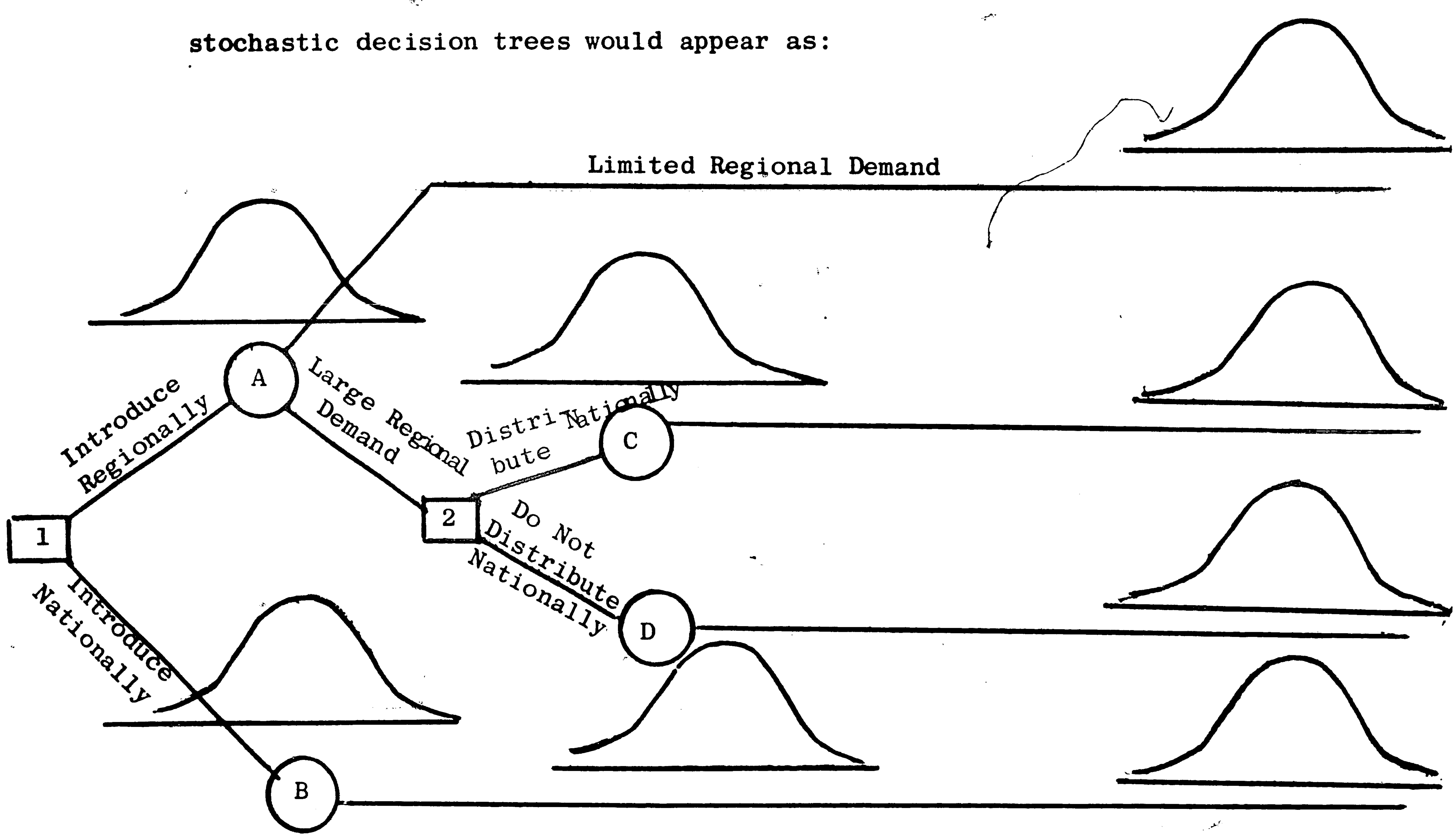

Figure 7

Simplified Stochastic Decision Tree

This stochastic method has an inherent investment risk analysis built into it. This is so, since risk analysis consists of estimating the probability distribution[1] of each factor affecting an investment decision; e.g., market size, investment request, operating costs, fixed cost, selling price, etc., and then simulating the possible combinations of the values of each factor to determine the range of possible outcomes and the probability associated with each possible outcome. Risk analysis can also be used for sensitivity testing of investment factors. In (59), Quigley and Hess illustrate how they have employed risk analysis on chemical industry investment problems by using the Monte Carlo method. It now appears that, "risk analysis is rapidly becoming an established technique in American industry. Several large corporations are now using various forms of the technique as a regular part of their investment analysis procedure", (58, p. 6). This impression is reinforced by David Hertz in (57), a well written article on risk analysis in capital investments.

3.6 Summary

Each method examined in this chapter has some advantages and some disadvantages. However, of all the methods proposed for the measurement of the profitability of capital investments proposals, the two most generally accepted are the present value and internal rate of return methods. This author prefers the present value method; especially since the rate of return technique can sometimes lead to ambiguous and/or indecisive results. But, another method, payback, has many sponsors and this popularity tends to raise it to the level of acceptance of the

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[1] Hertz in (57), and Hess and Quigley in (59), give recommendations for the techniques to be used to generate these distributions.

other two. Probably, the use of payback as an index of liquidity and "risk" accounts for its popularity. For investments made in very unstable foreign countries, this is no doubt the best method of evaluating investments. But, for many domestic investments, most decision makers, this author believes, have the wrong impression of the characteristics of the risk incorporated in the payback method. Payback measures only the risk associated with time since it assumes that a project continues uniformly with unchanged profits for a certain period of time and then suddenly ceases to be and ceases to have any value. It does not measure the risks associated with normal business dynamics; e.g., sales will decrease, costs increase, taxes increase, inflation increase, etc. Even if payback is used for time risk, it is still imperfect since it does not make allowances for the time costs of money nor the amount of the initial capital investment recovered. For example, if 95% of initial investment is recovered in the first one and one-half years and the remaining 5% in another year, it is obvious that most of the risk has been resolved in the first one and one-half years and it is incorrect to describe this project as one with a two and one-half years capital recovery period. Another problem with payback is the decision as to when to measure the beginning of the period of capital recovery. In some projects, long development phases are involved with small early expenditures. If one uses the commonly accepted starting point: the beginning of operations, then the pre-production period is ignored. Thus, two projects which have the same payback periods measured from the start of operations could have very different development and planning period such that the capital is at risk for much longer time in one case than in the other. Probably, the most significant period of risk is from the time of expenditure, not from the beginning of operations.

The present value method always gives the proper ranking of mutually exclusive proposals and the correct accept/reject decision for independent investments. Its big failing lies in the assumption of no risk or equal risks for all proposals. Yet, its very format makes it the ideal method for incorporating a factor for risk due to inflation. Since World War II, this factor has been of chief concern to management. This author recommends that the risk involved in inflation be treated as follows:

Let:

r = cost of equity capital in absence of inflation

s = cumulative rate at which the general level of prices is expected to increase

Then, without the effects of inflation, the present value of an amount A due in n years is:

$$P = \frac{A}{(1+r)^n}$$

However, if inflation is operative, then money values are not constant, and P does not truly indicate the present dollar equivalent of A due in n years. If s is known, then to translate A into present money worth, A must be reduced by the factor, $(1+s)^n$. Accordingly,

$$P = \frac{A}{(1+r)^n(1+s)^n}$$

$$= \frac{A}{[(1+r)(1+s)]^n}$$

Or, to generalize, the equity cost of capital in terms of r_i under inflation at rate s is:

$$r_i = (1+r)(1+s)-1$$

At this point the following should be noted:

a. The rate of inflation is multiplied by the equity cost of capital, and it is this product which is used to discount future payments. If both rates are low, i.e., less than 5%, their sum would be accurate enough for calculations.

b. If the general price level is expected to decrease rather than increase by the rate s per year, then:

$$P = \frac{A(1+s)^n}{(1+r)^n}$$

The stochastic tree method, because it inherently incorporates the risk dimension in the form of a probability distribution and the time discounting factor by use of the present value method, has accomplished a much desired marriage of risk and present value. If all the assumptions of the tree method, as based on Page 98 assumptions, are met, then it can be the best available operational method. Because of its very recent development and the requirement of a computer, little application studies have been conducted. The future may show this method to be superior.

CHAPTER IV COST OF CAPITAL

4.1 Cost of Funds

A very important factor[1] affecting investment decision making is the cost of capital to a firm. This cost depends upon its source. Although there are many ways to finance an investment; e.g., borrowing from banks, selling marketable securities, issuing bonds and/or stocks, using funds generated by operations, selling other assets or parts of its business, it is convenient to employ four general categories of fund sources: depreciation, retained earnings, debt, and equity financing. The first two sources of funds are internally generated while the last two are externally acquired.[2]

In most corporate enterprises, "capital expenditures are usually derived from internal sources. ...Consequently, the projection of the amount that can be expected from accumulated depreciation and retained earnings is usually the most important part of capital expenditure budgeting", (9, p. 38). A more detailed description of each fund follows:

A. Depreciation Fund

As a general rule, depreciation funds are always available and do not represent much of a problem so far as costs are concerned. Any

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1 Another important factor affecting investment decision making is the interaction of depreciation, taxes and profitability. "It has frequently been noted that the choice of depreciation method will affect the profitability of the investment." (4, p. 108). Two good references to show the importance of this interaction are (4, Chapt. 8) and (51).

2 Some sources, such as preferred stock, can be considered hybrid, since it has some of the characteristics of debt and equity securities.

firm will have the opportunity to reinvest these funds profitably. In fact, the cost of not reinvesting these funds and of not keeping the firm's capital investment intact can be prohibitive.

Many firms treat the depreciation fund as if it were in reality another equity fund. Merrett and Sykes (20, Chapt. 4), believe that such a concept is totally erroneous, and that the depreciation fund should be partitioned into debt and equity portions. The debt portion should consist of the money used for re-investment, since it obviates the need to repay a debt. Also, "when the opportunity cost of using part of the depreciation provision is effectively the repayment of debt, that part of the depreciation provision can meaningfully be regarded as attributable to debt and should be so regarded. Accordingly, it need only be invested at a rate of return sufficient to service debt capital", (20, p. 142). In other words, the cost of depreciation capital is to be considered as made up of the cost of the uses to which it could be put; e.g., to repay debts or distribute as dividends.

Prof. Eli Schwartz claims that where depreciation funds are the sole source of capital, the decision makers, "simply allocate the available volume of funds from depreciation to the highest yielding uses", (27, p. 246). For the growing firm which employs other than depreciation funds, "the cutoff point for new investments proposals will be determined, on the supply side, by the cost of the other forms of equity capital that are used to finance the expansion of assets", (27, p. 246).

It is generally agreed that depreciation funds should not be allocated according to where they originated. Instead, depreciation funds should be assigned to a general pool along with other capital funds. Any individual department of a company would then have to bid for use of these funds on its competitive merits.

B. Debt Fund

Some authors, such as A. J. Merritt and Allen Sykes (20, Chapt. 3), differentiate between short-term funds (trade credit, bank borrowing, bills of exchange, deferred tax payments), medium and long-term loans. However, for the purpose of financial analysis, the debt fund is usually associated with long-term debt, which are obligations the firm does not have to pay for a least a year.[1] Items that may be so classed are bonds, debentures, term loans, or in small firms mortgages on buildings.

The use of debt financing involves risks and imposes restrictions upon the freedom of management. Financing by means of debt implies an obligation to repay a firm's creditors at a fixed rate of interest in fixed periodic payments. Equity financing does not have these restrictions. Thus, according to the demands it creates, some companies try to avoid the use of debt.[2]

The cost of borrowed funds is the current effective interest rate. This effective interest rate may be different than the indicated or nominal interest rate of an outstanding debt security. For example, a

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1 They are also called funded debt or fixed liabilities. The portion of the long-term debt due within the current year is carried in the current liability section of the firm's balance sheet.

2 The effects of leverage, i.e., that amount of debt the firm uses in proportion to their own contributions in financing total assets, must also be considered.

bond issued at a discount or at a premium will have an effective interest rate different than its coupon rate.[1]

C. Equity Funds

Although equity or ownership of a corporation is legally tied to its common and preferred stock, for cost analysis purposes, preferred stock is defined as a form of debt and common stock as the equity fund. (The cost of using funds provided by an ordinary non-convertible preferred issue is simply the current dividend yield on such shares divided by the forecasted corporation profits tax rate. Earnings do not enter the calculations. For example, if the going dividend yield is 5% and the tax rate is 50%, the pre-tax cost of preferred stock financing is 10%. "The cost of preferred stock financing is relatively high compared to debt. On the other hand, as a subordinated form of financing - bearing some of the risks of equity - it helps to support the debt structure." (27, p. 141).)

According to Ezra Solomon, "the cost of capital derived from the issue of new common stock is a difficult concept and one about which little agreement exists in practice", (77, p. 242). Presently, there are at least four possible criteria in use for measuring the cost of these funds. These criteria are as follows:

Criterion 1: If these funds can be invested in projects whose rate of return is greater than that currently being earned on the

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1 The effective rate of interest for an outstanding issue can be determined by comparing the current market price for the security with the remaining payment obligations; e.g., for a bond, the effective interest rate is the rate of interest which equates the market price and the present value of the amount due at maturity plus the present values of the series of interest payments.

existing capital, then their benefits are greater than their costs. This will raise the company's average rate of return on its total investment. One of the fallacies with this reasoning is that the low profitability projects are given equal weight with the higher profitability undertakings in determining the firm's future investments.

Criterion 2: This criterion compares the anticipated rate of return of a new investment proposal with the present ratio of dividend payments to the market price of existing shares, i.e., to the dividend rate that the company expects to pay on new equity funds. The proponents of this method argue that any rate of return per share that is higher than the dividend rate per share will provide a net return to the firm.

Criterion 3: According to this criterion, the cost of equity funds is measured by the ratio of the current earnings per share to the current market price per share; i.e., the E/P ratio.

Criterion 4: This criterion is actually a refinement of the previous one. Instead of the current earnings per share (E), the numerator should measure the best estimate of the average of the future expected earnings per share if the proposed capital expenditures were not made. This measure shall be designated as E'. The correct index of equity costs is then E'/P. This measure, in the opinion of Ezra Solomon, is the only valid criterion for the cost of new equity capital.[1]

The following example of a company capitalized entirely by equity funds will be employed to defend this opinion:

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[1] Refer to (77, p. 243).

1.	Total book value per share	$ 30.00
2.	Total capitalization	1 million shares
3.	Current earnings per share (E/S)	3.00
4.	Current dividends per share	2.00
5.	Current market price per share (P)	20.00
6.	Best estimate of future annual earnings without new investments, e.g., an expansion project	3,300,000.00
7.	Best estimate of future annual earnings with the investment	4,200,000.00
8.	Salvage value of the project[1]	6,000,000.00
9.	Present outlay required for the investment	6,000,000.00
10.	Underwriting and flotation expense[2]	0

The proposed project, according to items 6 and 7, promises a return of $900,000 per year for an outlay of $6,000,000. The average rate of return is thus 15%. But, if the cost of capital were calculated in accordance with each of the four criterion, the following results are obtained:

Criterion 1: 10%
Criterion 2: 10%
Criterion 3: 15%
Criterion 4: 16.5%

Accordingly, the proposed project would be acceptable to Criteria 1 and 2, rejected by Criterion 4, and would be marginal according to Criterion 3. Which is correct? To finance the $6,000,000 project, 300,000 shares

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1 The unrealistic salvage value simplifies but does not affect the validity of the solution.

2 In a "real" case, these items, including any underpricing necessary to insure success of the issue, must be taken into account in computing the cost of new equity capital.

of stock must be issued at the current market price of $20 per share. The new earnings, with a capitalization of 1,300,000 shares will be about $4,200,000 or $3.23 per share. Without this investment, and without any new financing, the earnings per share would be $3.30 (items 2 and 6). Criterion 4 is the only one which correctly indicates that this investment would be unprofitable.[1] If P' indicates the amount of money the corporation actually receives for each share, then the correct measure of the cost of equity capital is E'/P'.

It must be realized that this cost model is restrictive because it implies:

> "A. Investment proposals within the company are homogeneous with respect to the quantity of yield offered.
>
> b. The company is financed entirely by equity funds.
>
> c. True earnings are equal to book earnings; i.e., the amount of depreciation deducted from the cash flow generated by operations is exactly enough to maintain earnings at the anticipated level.
>
> d. This level of earnings contains no upward or down trend", (30, p. 51).

Profs. Myron J. Gordon and Eli Shapiro in (56) have proposed a different method of evaluating the cost of equity. This approach described below, seems to have gained many followers:[2]

They (Gordon and Shapiro) have noted that both the dividend yield (Criterion 2) and the earnings yield (Criterion 3) fail to recognize

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[1] Criterion 3 gave marginal results only because of this particular example. It tends to give erroneous results, especially if small E/P ratios are involved.

[2] For example, Profs. Bierman and Schmidt base two chapters of their book on this method for determining the cost of equity.

that a share's payments can be expected to grow and in addition, that Criterion 3 fails to recognize that the corporation's earnings per share are not the payments made to the stockholder. "The practical significance of these failures is evidenced by the qualifications with which these two rate of profit measures are used by investment analysis", (56, p. 143).

To account for the propsective growth in a share's revenue, Profs. Gordon and Shapiro reasoned as follows:

Let, P_o = a share's price to t=o

D_t = dividend expected at time t

k = rate of profit or criterion

Then, the rate of profit on a share of stock is the value of k that satisfies

$$(1) \qquad P_o = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} .$$

But, it is mathematically convenient to assume that the dividend is paid and discounted continuously at the annual rates D_t and k giving:

$$(2) \qquad P_o = \int_o^{\infty} D_t \, e^{-kt} dt$$

To solve for D_t, two assumptions are made:

1. A corporation is expected to retain a fraction "b" of its income after taxes.
2. A corporation is expected to earn a return of "r" on the book value of its common equity.

Therefore, if Y_t equals a corporation's income per share of common after taxes at time t, the expected dividend at time t is:

$$(3)\quad D_t = (1 - b)\, Y_t$$

The income per share at time t is the income at (t-1) plus r per cent of the income at (t-1) retained:

$$(4)\quad Y_t = Y_{t-1} + rbY_{t-1}$$

or, if Y_t grows continuously at the rate ~~$g = br$,~~ we have:

$$(5)\quad Y_t = Y_o e^{gt}$$

Hence, from equations 3 and 5:

$$(6)\quad D_t = D_o e^{gt}$$

If this expression is substituted for D_t in equation 2 and integrated, it yields:

$$(7)\quad P_o = \int_o^{\infty} D_o e^{gt} e^{-kt} dt$$

$$= D_o \int_o^{\infty} e^{-t(k-g)} dt$$

$$= \frac{D_o}{k-g}$$

If $k > g$, a condition easily satisfied, for otherwise P_o would be infinite or negative:

$$(8)\quad k = \frac{D_o}{P_o} + g$$

In words, this means that the rate of profit or the cost of equity at which a share of common stock is selling is equal to the current dividend divided by the current price (Criterion 2) plus the rate at which the dividend is expected to grow.

For example, if the current dividend is \$6.00 per share, the current market price is \$150.00 per share, and the dividend per share is expected to increase at about 2% per year, the formula gives:[1]

$$k = 0.04 + 0.02 = 6\%$$

D. Retained Earnings

The dividend policy of a firm has an important effect on the amount of retained earnings of that firm. For example, the firm may try to retain all funds that can be used profitably, i.e., their rate of return exceeds their costs. In this case, retained earnings would be a fluctuating residual left over from earnings. Or, the firm may try to retain a certain percentage of earnings for contingencies and growth. In this case, the dividends would be the residual, and their magnitude would vary with earnings.

In order to arrive at a correct measure of the cost of retained earnings, Ezra Solomon in (77, p. 245), proposes that these earnings must be thought of as analogous to new equity funds. It is then assumed that the company paid out these funds as dividends and that the stockholders then reinvested the funds in the company. On this basis, the correct measure of the cost of retained earnings would be the same as that for equity funds: E'/P. The P rather than P' is used because no floatation costs, etc., would be involved in obtaining these funds.

Stockholders' personal income taxes may be an important factor in determining whether to retain earnings or not. If the rate of return

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[1] Adjustments must be made for expected stock splits and stock dividends.

that can be earned on these funds is less than their cost, then they should be paid to the stockholders. If there were no income tax on dividends, then E'/P would be the correct measure of their cost. With taxes, however, this ratio must be adjusted to $\frac{E'}{P}(1-m)$ where m is the common tax rate. Thus, it might be more profitable from the stockholders' viewpoint for the company to reinvest the funds at a lower rate than that at which the stockholders could reinvest them. Although the stockholders might reinvest these funds at a higher rate than the corporation, they would have a smaller amount to invest after taxes.[1]

To illustrate the above, let's assume that a company has $1,000 either to reinvest or to distribute to stockholders as dividends. The company can reinvest these funds at 10%. The stockholders can reinvest the funds in the market at 20% (E'/P), but they are subject to a 50% tax. The company can earn $100 for their stockholders. The stockholders would also earn a net amount of $100. In this case, the stockholders would be just as well off if the company retained the money. For any rate higher than 10%, the company could earn more for its stockholders than the stockholders could earn by investing for themselves.

If the stockholders were all subject to the same tax rate, the correct measure of the cost of retained earnings would be $\frac{E'}{P}(1-m)$. In reality, the personal tax on the company's shareholders is not uniform; e.g., tax-free institution and top income groups in society may be shareholders. There is no way to adjust for this multiplicity of tax

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1 "The fact that stockholders are not all in the same marginal tax bracket creates a difficult problem, and one for which there is no solution at either the conceptual or the operating level." (77, p. 246).

rates. Therefore, either the tax rate must be ignored or some kind of average tax must be assumed.

4.2 Effect of Corporate Income Tax on the Cost of Capital

Corporate income tax is an important consideration in determining which type of financing is to be used. This tax has the effect of lowering the cost of debt relative to equity financing. The reason for this is that interest payments are tax deductible while dividend payments are treated differently.[1] Part of the debt cost, therefore, is not really a cost, since it would have been lost through taxes. To illustrate this: Assume that a corporation will acquire $10,000 by either debt or equity financing, that the corporate tax rate is 50%, and that for simplicity sake, the current yield demanded by the market is 5% for both debt and equity funds. If the firm decides upon debt financing, it must be able to earn at least $500 a year to avoid default. This would require an earning rate of 5% on the $10,000. If equity financing is chosen, this rate of return would be inadequate. The corporate tax would reduce the $500 earnings to $250. The resultant E/P yield would be only 2.5%. To justify financing on equity the company would have to earn at least $1,000 before taxes. This would be equivalent to a return of 10% on the original $10,000. With a corporate income tax of 50%, therefore, a company must earn about twice as much with equity funds as with debt funds.

Obviously, then, if the cost rate of debt and equity funds are to

[1] The corporation is taxed on its earnings. The stockholders are then taxed on that proportion of a company earnings distributed as dividends.

be compared, they must be put on the same basis. When the corporate tax is considered, the cost of debt funds (r') is as follows:

$$r' = \frac{\text{Effective Interest Rate}}{1 - \text{Corporate Tax}}$$

This calculation then helps to resolve the problem of measuring and comparing the cost of individual sources of funds, but it does not clarify what should be done when the total cost of capital is a composite of more than one type of fund.

4.3 Composite Financing

The question of how to treat the cost of capital when more than one type of fund is used for financial budgeting has been and remains one of paramount importance to economists, financiers, and decision makers. Although the solution has been formulated in several ways, as yet no clear-cut, generally accepted solution exists. "This gap represents the weakest link in the theory of capital budgeting, and, until it is filled, capital budgeting theory will remain, at best, only a partial guide to decision making in this important area of business activity." (77, p. 241)

Six of the proposed solutions will be described, in order to demonstrate the current theoretical thinking about this problem. Most of these methods ignore depreciation funding, because a successful firm is supposed to have little real choice about reinvesting such funds.[1] Merrett and Sykes, in what is probably one of the most respected books on finance and capital budgeting in Great Britain, (20, Chapt. 3), refuse

[1] Typical examples are (4), in which the authors discuss the cost of capital in two chapters but never mention depreciation funding; also Joel Dean's discussion in (9, Chapt. 3), and (70), (56), (50), and (75).

to recognize such an elimination of depreciation funding. To them, depreciation funds should be divided into its debt and equity components for proper calculations of its cost. In this fashion, it takes its place together with debt, equity, and retained earnings in any composite cost calculations. However, most authors believe that omitting depreciation funds, even if it has inherent opportunity costs, is not a serious flaw. They claim that the practical advantages of ignoring the cost of depreciation funds far outweigh the relatively insignificant theoretical loss. Thus, when depreciation is ignored, the problem resolves itself into one of finding the combined costs of debt and equity with the latter being composed of retained earnings and new equity funds.

Proposed Solution 1:

This method employs what is probably the most obvious technique: the use of a weighted average of the costs of the debt and equity funds. The weights to be used are the proportion of each type of fund in the company's capital structure.[1] This method can be illustrated by the following tabulation:

	Proportion of Capital Structure	Cost of Each Type	Weighed Cost of Each Type
Debt	40%	5%	2%
Equity	60%	10%	6%
Total Cost	---	---	---
of Capital			8%

[1] This method is employed by some state commissions in the regulation of public utilities.

One criticism of this approach is that the existing capital structure represents the past financing of the company. Perhaps, the proportions of debt and equity in the anticipated future financing should be the weight applied. But, here again, we are entering into areas of uncertainty. The overall simplicity of this technique is most appealing.

Proposed Solution 2:

This solution is closely related to Proposed Solution 1. In this method, the cost of capital is also defined as a weighted average of the cost of each type of capital. But, the weight for each type of capital is the ratio of the market value of the securities representing that source of capital to the market value of all securities issued by the company. The term security includes common and preferred stocks and all interest-bearing liabilities including notes payable. For example, suppose the market value of a firm's common stock is estimated at $45 million. The market value of its interest bearing debt is estimated at $30 million, and the average before tax yield on these liabilities is 6% per year, which is equivalent to an after tax basis of 2.88% per year (6 x .48, assuming a 52% tax rate). Assume the company is currently paying a dividend of $8.00 per year and its stock is selling at $100. The rate of growth of the dividend is projected to be 2% per year. Hence, the average cost of the common stock equity is:

$$k = \frac{\$8}{\$100} + 0.02 = 10\%$$

Therefore, the average cost of capital for the company as a whole is computed as follows:

Capital Source	Proportion of Total Capital	~~Cost~~	Weighed Cost
Equity	45/75 = .60	.10	0.06
Debt, interest bearing	30/75 = .40	.0288	0.012
Average Cost of Capital			7.2%

Proposed Solution 3:

This technique assumes that all funds, regardless of their source, have the same cost as debt funds. The fact that the cost of debt funds is fairly easy to obtain seems to be a point in favor of this method. But, "because it ignores the cost of equity funds, this approach provides a wrong criterion for judging the acceptability of investment proposals. . . . Whenever the net cost of equity funds is higher than the net cost of debt funds, (and this is the usual case), the use of this approach will lead to the acceptance of proposals that should properly be rejected", (77, p. 248). The following example will serve to reinforce this opinion:

A company has $300 as the maximum equity funds available from retained earning. Assume that for each dollar retained it can borrow a dollar at the net interest cost of 2%. Thus, the company has a maximum borrowing capacity of $300. Also, assume that the current earnings yield in the market at which a stockholder could reinvest any dividends is 10%. The company is considering three alternative investment projects:

1. $200 at a rate of return of 14%

2. $200 at a rate of return of 7%

3. $200 at a rate of return of 3%

The cost of capital (2%) is less than the rate of return on any one of these proposals. Accordingly, the company, by this proposal, should borrow the required $300 and invest in all three projects; i.e., $300 is retained and $300 is borrowed. But, if this were done, the total yearly earnings for the stockholders would be $48; i.e., ($28 + $14 + $6). After paying interest on the borrowed funds, $42 would remain. This, actually, is not in the best interests for the stockholders. To demonstrate this, only the first two proposals will be accepted and $400 would be invested; $200 from retained earnings and $200 from borrowing would finance this investment, and $100 would be paid out as dividends. This action is based on the fact that only the first proposal justifies retaining the earnings within the company; while the second proposal exceeds the cost of borrowed funds. The third project also exceeds the borrowing cost of funds, but the company has already reached its borrowing limit. The net results of this operation would be as follows:

$200 at 14%------------------	$28
$200 at 7%------------------	14
Gross Earnings---------	$42
Less Interest Costs	4
Net Earnings	$38
Plus earnings from Dividend Reinvested at 10%---------------------	$10
Total Earnings	$48

This investment activity yields the stockholders $6 more than the previous plan. This method is then inadequate.

Proposed Solution 4:

This approach identifies the cost of equity funds as the relevant cost of all funds regardless of their source. The "logic" behind this proposed solution is as follows: In addition to their monetary costs, debt funds also have a non-monetary cost; e.g., constraints placed on the borrower by the terms of the loan, and the risk of default together with its consequences. "Companies will finance by debt so long as the total real cost of the increment of borrowing is below the cost of financing by new equity funds." (77, p. 249). But, as the firm continues to borrow, the real cost of debt funds increases, because the lender will set higher nominal costs and also because the risk incurred by the borrower will rise as the proportion of debt to equity rises. At the time, when borrowing costs have reached equity costs, the company will turn from debt financing to equity financing. Thus, it can be assumed that there will be a continual adjustment of a firm's capital structure so that the cost of its debt and equity funds are always in balance. Since it is usually impossible to give quantitative values to the non-monetary costs of debt, the cost measurement is made on the equity funds.

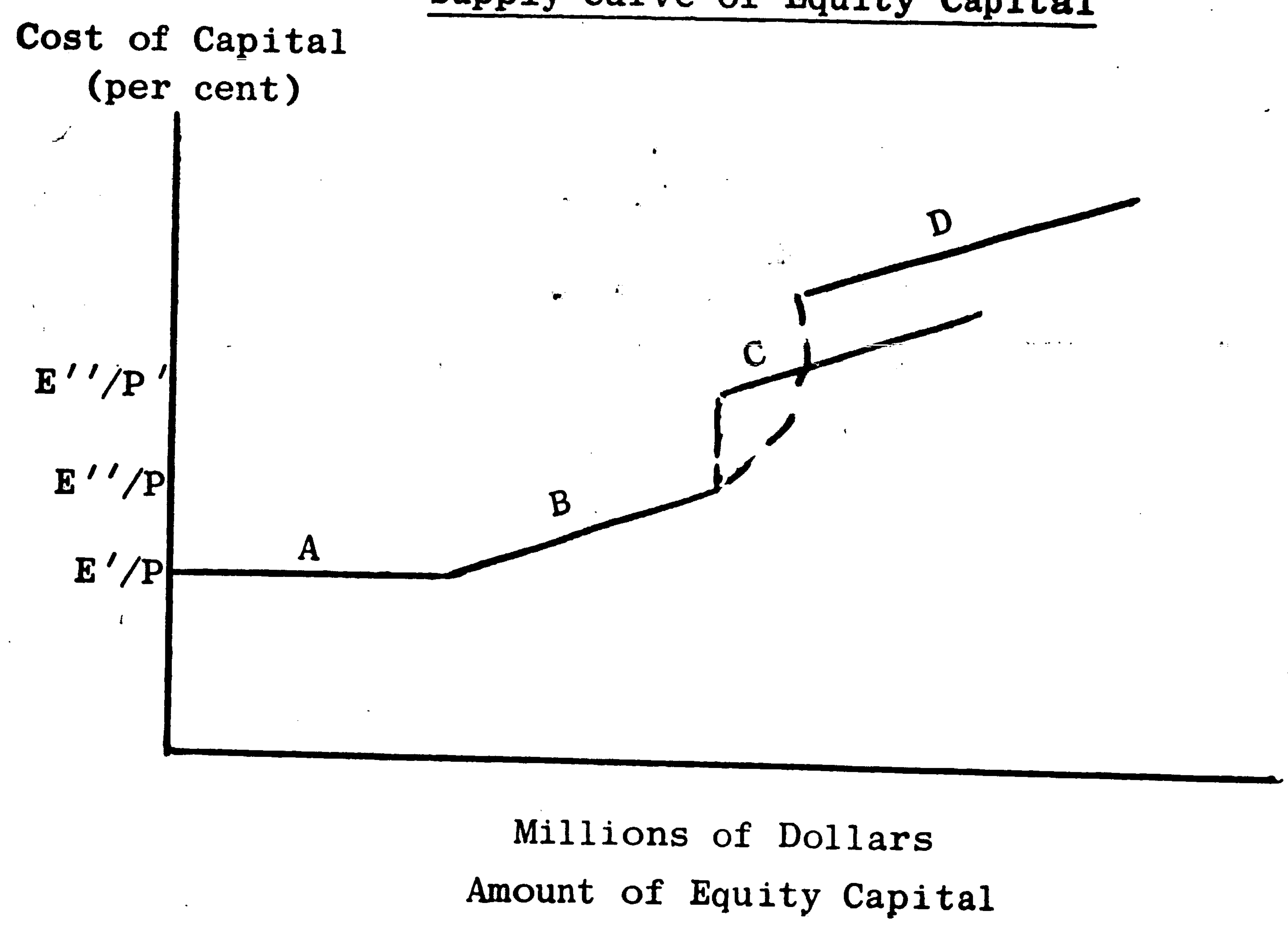
The major problem with this approach is the fact that it does not provide a guide as to how a firm can achieve a balanced capital structure. Without such guidance, this method must be viewed as having little practical value, although it does attempt to take borrower's risk into account.

Proposed Solution 5:

This method, which is strongly endorsed by Ezra Solomon, considers debt funds as a form of quasi-equity funds. It is easier to understand this concept if we view the equity capital supply versus the cost of capital, initially without borrowing. Figure 8 is a supply schedule of equity funds.

FIGURE 8

Supply Curve of Equity Capital



In the above representation, "A" is assumed to be the depreciation funds, but no attempt is made to measure their cost explicitly. Part "B" of the curve represents retained earnings. The initial cost of these funds is E'/P as was previously explained. As more and more dollars are reinvested, the earnings are assumed to rise faster proportionately than does the price. The slope of this part of the supply curve, therefore, is upward. The cost of the last unit of available retained earnings is E"/P where E" is equal to the expected earnings per share, including the earnings expected from projects to be financed from depreciated allowances and all of retained earnings. Part "C" of the curve represents new equity funds. The abrupt rise between "B" and "C" is caused by the fact that E"/P changes to E"/P'. The P' amount is less than P (actual market price) by the amount of underwriting fees, floatation expenses, etc. Accordingly, the value of the total cost ratio rises sharply. Part "D" shows what would happen if the company reduced dividends below past levels. "Such action is likely to bring about a rapid rise in the cost of funds because it will generally reduce the market price of stock", (77, p. 247), and cause the slope of the retained earnings curve to increase noticeably. Actually, Part "D" then, represents the new equity financing at this higher level of stock prices.

At this point, how can this curve be modified so that it can be applicable to a company that uses debt funds. It is possible to consider the use of debt funds only because of the equity capital already invested in the company's assets.[1] When a company borrows money it frees some of its equity funds that had previously been tied up in its asset structure. In other words, "the supply of usable equity funds can be derived not only from current earnings and new stock floatations but also from borrowing on the strength of these real assets", (77, p. 251). These newly freed equity funds are then added to the supply schedule described above. If the borrowing has no influence on the

[1] Ezra Solomon refers to this as the "general borrowing power of equity", (77, p. 251).

price of the company's stock, the net effect is to stretch out the supply curve to the right. In effect, the debt funds have become quasi-equity funds.

In using these methods, each new proposal is given a borrowing quota. This quota is based on the minimum amount of earnings expected from the proposal so as to prevent the possibility of default on the loan. When this is done for all proposals, the amount of total borrowed funds is known. The amount of new equity financing required is simply the difference between the total cost of the investment and the total borrowed funds. This technique allows the decision maker to formulate a schedule that shows, for each proposal, the amount of equity funds it needs and the net rate of yield it promises to return if these funds are granted for it. Thus, in addition to the borrowing power the company possesses due to its existing assets, it will add new potential borrowing power for each new proposal that is accepted.

Proposed Solution 6:

This method has been sponsored by Profs. Modigliani and Miller in (70).

Let

C = cost of capital

X = corporate profits before interest payments

D = total market value of the company's debt securities

S = total market value of the company's equity securities

$V = S + D$ = total market value of company

Then, if it is agreed that "the market value of any firm is independent of its capital structure and is given by capitalizing its expected return at a rate appropriate to its class", (70, p. 156), we can let:

$$C = \frac{X}{D + S} = \frac{X}{V}$$

The value the market places on the firm's securities is supposed to reflect the total capital value of the firm at any particular time. In return for this capital contribution, the security holders have received claims to the total value of the company's earnings (X). The ratio of what the corporation has given up (X), to what it has received in return (D + S) is, therefore, the correct measure of the cost of capital. Prof. David Durand criticized this proposal in detail in (50).

The above methods should give some idea of the complexity of this problem of composite financing. Perhaps this can be best appreciated by the following comments by Profs. Bierman and Smidt, (4, p. 140): "A person who desires neat solutions with one correct answer should avoid the computation of a corporation's cost of capital. Usually such an answer is impossible. If ten experts independently computed the cost of capital of the IBM Corporation, there would be ten answers. Although an exact answer to a corporation's cost of capital is elusive, it is possible to establish a range which, with a high degree of probability, includes the cost of capital."

4.4 Effect of Capital Structure on the Cost of Capital

It is possible to view the effects of capital structure on a firm's cost of capital from two entirely different approaches:

a. The Traditional Approach

This point of view, most commonly accepted, holds that the moderate use of debt lowers the total cost of capital. The basis for this premise is the "belief" that debt is much cheaper than equity capital. This can be illustrated by a comparison between a firm's interest yield on bonds and the earning yield on stock. Stock yields are usually several times larger than bond yields.

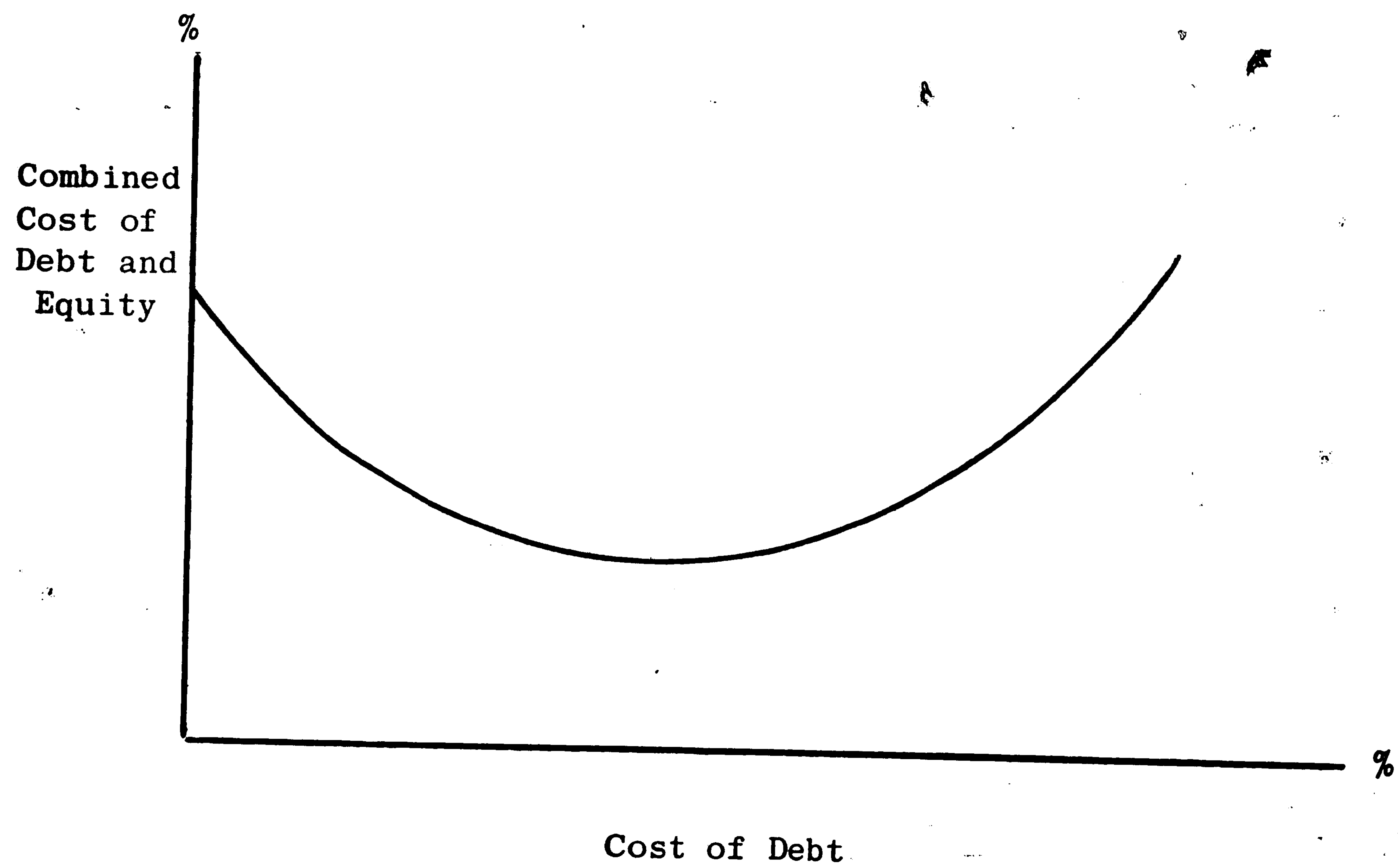
The cost of debt funds, however, is not a constant. As the debt ratio increased, the company would have to offer higher interest rates to compensate bondholders for their increased risk. In addition, the greater proportion of debt in the capital structure would decrease the value of the common stock. This means that the cost of equity funds also increases with increasing debt ratios. The use of debt financing, however, tends to reduce the total cost of capital up to a point. Evidently, there must then exist an optimum combination of debt and equity capital. This can be illustrated in Table V, where the combined cost of capital is calculated by weighing the cost of each type of fund by their respective proportions in the company's capital structure. This table indicates that for the year under consideration (1937), the ideal utility capitalization consisted of approximately 30% bonds and 70% common stock.

TABLE V

Effect of Capital Structure on the Total Cost of Capital - An Example From Utility Companies[1]

Percentage of Debt in Capital Structure	Cost of Debt (%)	Percentage of Equity in Capital Structure	Cost of Equity	Combined Cost (%)
0	0	100	6.56	6.56
10	3.15	90	6.84	6.12
20	3.21	80	7.19	5.76
30	3.35	70	7.59	5.50
40	3.70	60	8.17	5.51
50	4.60	50	9.37	6.17

Graphically, the relationship between debt funds and combined funds can be represented by Figure 9.



Cost of Debt vs. Combined Cost of Debt and Equity

FIGURE 9

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1 Source is (8, p. 232) - This table is based on a survey of telephone, electric and gas utility companies for the year 1937.

Table V and Figure 9 demonstrate that an optimum capital structure does exist; i.e., the structure at which the combined costs are a minimum. "It is impossible to give any simple rules for determining in advance the optimum capital structure for a particular firm. Theoretically, the optimum structure is reached when an additional debt issue, in substitute for stock equity, will result in a decrease in the price per share of the common stock." (4, p. 159).

b. Modigliani and Miller Approach

Profs. Modigliani and Miller, in 1956, in a paper delivered at the annual meeting of the Econometric Society, presented their three propositions which contradicted the traditional approach. The theory put forth by Profs. Modigliani and Miller, however, has not been eagerly endorsed by many theorists. In fact, Prof. David Durand has vigorously attached the Modigliani and Miller (henceforth referred to as M and M approach) in (50). More recently, Alexander Barges won the 1962 Ford Foundation Doctoral Dissertation Award for his unfavorable analysis, (mostly statistical), of the M and M philosophy (2). And in 1963, Ezra Solomon attempted to disprove the M and M approach through logic and theoretical analysis in (30, Chapts. 8,9). M and M take as a starting point a world of perfect markets in which all investors act rationally and in which all types of investors are not handicapped in their actions. Given this, M and M assume that firms can be divided into homogeneous classes in accordance with business risk or the certainty of their anticipated income stream. In other words, the income generated by the assets of any one firm in a class is subject to the same degree of uncertainty as the income generated by the assets of the other firms in that class. Having thus assumed perfect markets and classes of homogeneous firms, M and M then present their propositions.

Proposition 1: The average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class.

To express this proposition in equation form, let:

X_j = the expected future profits of company j in class k before deduction of interest.

S_j = the market value of the common shares of company j.

D_j = the market value of the debts of company j.

V_j = the total market value of company j.

P_k = the expected rate of return on the common stock of an unlevered company in class k.

$\bar{X}_j/V_j$ = average cost of capital of company j.

Then, if

$$V_j \equiv (S_j + D_j) = \frac{\bar{X}_j}{P_k}$$

for any firm j in class k,

$$\frac{\bar{X}}{(S_j + D_j)} \equiv \frac{\bar{X}_j}{V_j} = P_k$$

This relationship indicates that the firm's cost of capital is dependent on its total market value (V_j), but is independent of the composition of that market value. Thus, a company that has the same earnings (X_j) and the same total market value (V_j), is supposed to have the same cost of capital whether it be 10%, 40%, or 80% financed by debt.

Proposition 2: The expected yield of a share of stock is equal to the appropriate capitalization rate P_k for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between P_k and r (the interest rate).

M and M define the expected yield of a share of stock as the expected earnings yield, and not dividend yield. Also, M and M define debt to include all securities senior to the common stock.

Proposition 3: The cut-off point for investment in the firm will in all cases be P_k, and will be completely unaffected by the type of security used to finance the investment.

Mathematically, this proposition can be formulated as follows:

$$i_j = P_k + (P_k - r) \frac{D_j}{S_j}$$

where: i_j = expected rate of return on the common stock of company j.

r = the rate of interest.

The mathematical proofs given by M and M have been challenged by Alexander Barges who has demonstrated that, "with respect to the empirical methods employed by M and M it was found that under very frequently encountered conditions, their methods will result in tests which are biased against the traditional views", (2, p. 101).

M and M's basic premise of a perfect market is suspect since a perfect market is defined as one in which two identical commodities can not sell at two different prices. Further, their assumption about the possibility of separating firms into equivalent return classes seems unrealistic. Also, the development of the M and M approach relies heavily on an arbitrage process which ignores the differences between corporate debt and personal debt. In other words, it assumes that personal and corporate leverage can be regarded as equivalent; because for certain conditions a stockholder in a levered firm will presumably transfer the company's leverage to himself by selling the stock, taking out a personal loan, and investing the total preceeds in the stock of an unlevered firm. The fact that the stockholder is liable for the full amount of his personal loan when he levers himself, compared to his limited liability when he held stock in a levered corporation, is completely ignored. These and other assumptions, (the effects of taxation are ignored, and the yield curve is supposedly the same for all borrowers), probably accounts for the general unacceptability of the M and M approach.

4.5 Summary

The basic overall capital investment and/or budgeting problem is one of supply and demand. The demand that a proposal can exert for funds is subject to its own (the proposal's) profitability. The supply of funds depends on their cost. If these curves are plotted (Capital Funds vs. Cost of Capital), their intersection should result in a unique, theoretically correct solution to this capital budgeting problem.

If the two main methods of evaluating the capital investment problem are viewed in the light of the above statement, a difference appears. The present value method has within its mechanics both supply and demand factors. It can, however, be argued that the supply factors are imperfect, since more than one value for the cost of capital usually exists. In fact, it generally varies with the amount and type of funds used. The rate of return method is claimed by many to be evaluated in relative isolation from the cost of capital. This claim, is believed by this author, to be unrealistic and has been expanded upon earlier in this thesis. However, operationally, most decision makers treat the choice proposals based on rates of return as follows:

(a) Evaluate all proposals for their anticipated rate of return.

(b) List the proposals in order of magnitude of their rate of return.

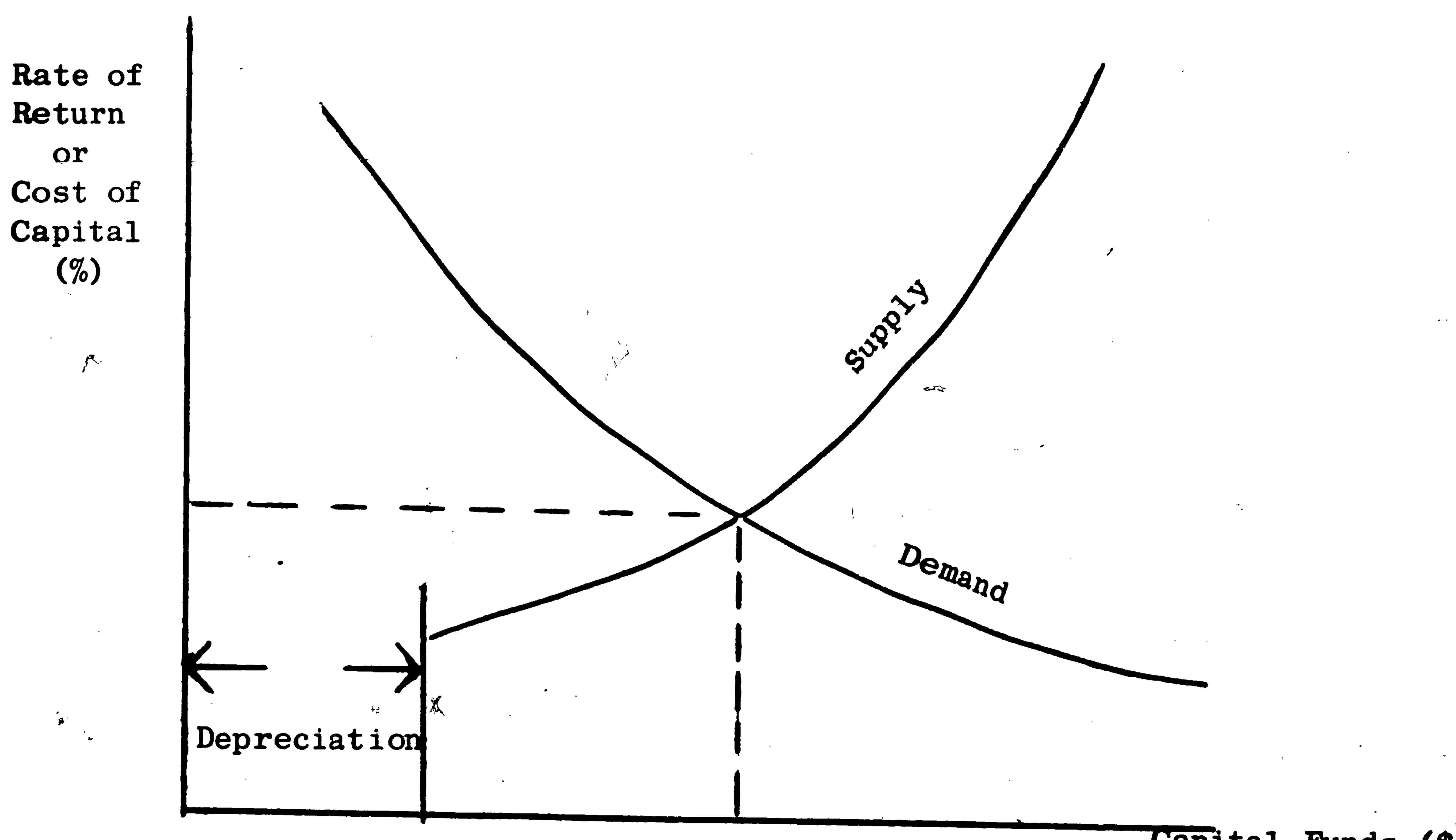
(c) Generally, a cutoff rate is selected; e.g., 15%, and any investment whose prospective rate of return is below this is "inadmissable".

(d) Develop a cumulative demand table which includes each proposal based on the above ordering.

(e) Act on all proposals which are included in this cumulative amount which in turn is equal to the amount available or to be made available.

The decision maker believes that by evaluating proposals with the above techniques he does not concern himself with a cost of capital. At any rate it is considered in a separate evaluation similar to the schedule developed for the fund demand, when the time comes to raise capital.

In reviewing the supply and demand of capital investments, it should be noted that the cost of capital determination is the second weakest link in the whole chain of capital budgeting. (The first weakest link is the uncertainty of the future.) There is no general agreement as to how the cost of capital should be determined. None of the methods previously described are completely acceptable. But, to function, every firm must determine the proposals which are to be considered, acted upon, and the amounts and composition of the necessary capital funds and their respective costs. If all such information were available to the firm, its capital budgeting problem can be easily solved by simply plotting the supply vs. the demand, and extrapolating the point of intersection horizontally and vertically, to identify the total amount of capital funds and the potential earning rate, (since rate of return and cost of capital would both represent the same axis):



Supply and Demand Schedule for Capital Funds

FIGURE 10

But, by its imperfection, such a model only approximates the real world. Until the perfect model is developed, the decision maker will fill in the gaps with his rules of thumb, experience and intuition. The closer to ideal and model becomes, the less he will use these very personal aids.

CHAPTER V UTILITY AND ITS ROLE IN CAPITAL INVESTMENT

For "problems arising under risk conditions, our analysis will reveal that an optimal decision can be defined as one which maximizes the mathematical expectation of value or utility", (1, p. 32). Accordingly, a review of the role of utility theory in investments is pertinent.

"Relying on Jacques Bernoulli's famous Law of Large Numbers, theorists have for nearly two and one-half centuries (Ars Conjectandi was published in 1713) attempted to justify expected return maximization in virtually any setting through the purely formal argument that, in the long run, 'the expected' becomes 'the certain'." (11, p. 8) But, to be theoretically valid, expected return maximizing must meet the following criteria:

1. "Experiments" involved must be repetitious in nature.
2. Successive observations are independent of their predecessors.
3. Possible outcome are not extreme.

However, these three factors do not describe any capital investment environment. It is fairly obvious why the first two are foreign to such an environment and no discussion is warranted. The third criterion is usually difficult to adhere to in investment decision making. A rational investor's concept about extreme values can best be demonstrated by the St. Petersburg Paradox.[1]

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[1] An excellent mathematical treatment of this subject is given by Prof. Paul A. Samuelson, in "The St. Petersburg Paradox as a Divergent Double Limit", International Economic Review, Vol. 1, No. 1, Jan., 1960.

This paradox can be described as follows:

A person buys a chance to flip a coin until heads appears. Should it appear on the first throw, he receives $1. Should it appear on the second, third, fourth, ..., n-th throw, he receives $2, $4, $8, ..., $\$2^{n-1}$, respectively. How much should he reasonably pay for a chance to play the game?

The expected value of this game, assuming a fair coin, can be computed as follows:

$$E(V) = \sum_{n=1}^{\infty} (\tfrac{1}{2})^n 2^{n-1} = \tfrac{1}{2} \sum_{n=1}^{\infty} 1^{n-1} = \infty$$

Accordingly, a person should be willing to pay an infinitely large sum of money to play the game. But, this does not seem to be in accordance with the rational behavior of reasonable people. Most of these persons would reason that the bank itself does not have infinite resources. Any throw beyond the consecutive toss that could break the bank is then meaningless. Also, very long runs of heads or tails are very improbable.[1]

The decision maker does not usually treat extreme values in the same linear relationship relative to risk and uncertainty as he does an average value. A loss of $100,000 is not 10 times as bad to a firm as a loss of $10,000 especially if any loss over $10,000 would mean bankruptcy.

Thus, because most investments are one shot affairs, dependent,

---

[1] For example, $P_{20\ heads} = (\tfrac{1}{2})^{20} = .95(10^{-33})$

and usually involve large sums of money, profit maximization,[1] cannot be regarded as an overall investment model under uncertainty.

> "There are a wealth of arguments to the point that a simple theory of profit maximization will not in fact predict the choices made by managers. While profit can be made operational, it represents one with difficulties in the process. One of the most troublesome of these is the allocation of overhead, which requires judgment on the part of the calculator of profit. Many other such judgments are required to produce an operational definition, giving it a rather arbitrary character. One must couple with assertions of the profit motive the answers to several difficult questions:
>
> a. How far in the future are profits to be calculated in a given decision? How are future profits to be related to present profits?
>
> b. What is the relation between profit and risk or uncertainty? How is one to combine profits and probabilities in making a choice?
>
> c. What explanation is to be given for the three following instances of management decision-making behavior?:
>
> 1. Managers do not select the profit maximizing alternative and engage in policies apparently antithetical to the maximization of profits.
>
> 2. It is possible to show a manager ways of making more profit which he will decline.

---

[1] "Recent dissatisfaction with the profit maximization assumption has been one of two types. Either it is claimed that maximization is a non-operational concept and hence a different analytical framework is required or the exclusive attention to the profit goal is disputed." (37, p. 6). Prof. Williamson then attempts to demonstrate the importance of managerial motivation; i.e., salary security, "dominance", professional excellence, and discretion in developing a model of the business firm's behavior in seeking profit maximization. J. M. Clark sums up his opinions in (7, p. 91): "The assumption that a firm pursues maximum profits is an extreme simplification. Indeed, it is a simplification to assume that any unified objective governs all the operations of a firm, especially a large one. This covers up a multitude of divergencies. Theory has a tendency to dispose of these as being either differences of view on what kind of action will maximize profits on a given case, or differences between longer or shorter time perspectives in which profits may be viewed."

3. Managers explicitly use other goals than profit in their work. For example, they are concerned with such measures as sales volume, rate of growth, share of market, and industry position---and good corporate image, satisfactory labor relations,[1] job security, and satisfaction and public service." (21, pp. 159, 160).[2]

Concerning the use of money as a valid guide to action, Prof. Schlaifer claims: "Expected monetary value should be used as the decision criterion in any real decision problem, however complex, if the person responsible for the decision would use it as his criterion in choosing between (1) an act which is certain to result in receipt or payment of a definite amount of cash, and (2) an act which will result in either the best or the worst of all the possible consequences of the real decision problem." (26, p. 29). Prof. Schlaifer "proves" this rule by showing that decision makers who do not follow the rule will make choices which are logically inconsistent.

However, he agrees that this rule generally becomes inoperative when the amounts involved are quite large.

As an alternative to expected profit maximization, most theorists have adopted the principle of maximizing expected value. Prof. Ackoff justifies this principle by its "apparent reasonableness".[3] To them,

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1 For example, an investment in recreational facilities, building and swimming pool for employees.

2 Prof. Morris also lists, in detail, five arguments usually given for the use of the profit maximization premise.

3 (1, p. 38).

it just seems obvious that rational[1] decision makers ought to function within the framework of such a principle.

The original proposal for a principle of maximizing expected utility was developed by Daniel Bernoulli, as a result of his investigation of the St. Petersburg Paradox. He assumed that utility followed a function that more than a century later was proposed by Fechner for subjective magnitudes in general and is now called Fechner's Law. Bernoulli claimed that an individual's marginal utility of money will always be inversely proportional to the total amount of his wealth. In effect, he expressed the total utility of money as a logrithmic function: If wealth increases from x to x + y, there is an increase in utility which is

(a) Proportional to the increase y

(b) Inversely proportional to the initial wealth x.

This means that:

$$K \frac{y}{x} = \mu(x + y) - \mu(x)$$

or

$$\frac{K}{x} = \frac{\mu(x + y) - \mu(x)}{y}$$

Letting

$y \to 0$, one obtains

$$\mu'(x) = K \frac{1}{x}$$

---

[1] A whole chapter, Chapter 8, is given to this topic of rational behavior in C. West Churchman's book (6). The author defines rational behavior in terms of a set of axioms. A psychologist states: "In more complicated choices the measured utilities can be used to calculate what response the person should make to maximize his expected utility. This response is the rational choice." (24, p. 61).

or

$$\mu(x) = K \log x + c$$

Thus, if a firm has an initial wealth of s, it should prefer the prospect which maximizes the sum:

$$\sum_i f(x_i) \log (x_i + s).$$

Bernoulli called this sum "moral expectation", as distinct from mathematical expectation:

$$\sum_i x_i f(x_i)$$

The principle of maximization of expected utility, however, was adopted by the theorists, only after the concept of constant utility or indifference mapping,[1] which incorporated expected value preference and risk aversion,[2] had failed to adequately explain real life situations. Because economists are still working with this indifference concept, it is interesting to investigate[3] it:

Let

$$v = f(\mu, \sigma) \text{ where}$$

v = investment's certainty[4] equivalent

---

[1] An excellent discussion of Indifference Curve Theory is given by Prof. Milton Friedman in (12).

[2] Risk aversion; i.e., that the utility of money must be represented by a concave function, was originally proposed by Alfred Marshall. Prior to his theory, Adam Smith had reasoned that most people had a risk preference; i.e., that their "attitude to risk" had to be represented by a convex utility function (increasing marginal utility of money). Refer to: Marshall, A., Principles of Economics, London, 1890.

[3] Another reason to investigate this concept is due to the fact that H. Markowitz used it as his base for generating a model for stock portfolio selection. Refer to (17).

[4] The certainty equivalent is the amount which involves no risk or uncertainty. It can also be derived through lottery techniques. An excellent reference on this technique is (26, Chapt. 2).

μ = the mean of alternatives

σ = the standard deviation of these alternatives

Then define a function:

$$v_o = f(\mu,\sigma)$$

which can be equated to an indifference curve which is defined by the locus of μ,σ points whose values are deemed to be equal to a certain return of v_o; i.e., the same utility.

Each alternative can be similarly defined:

$$v_i = f(\mu_i, \sigma_i)$$

Now, it is a generally accepted fact that most investors will desire an increase in μ and would tend to disapprove of alternatives with increasing σ. This expressed on a given constant utility curve is:

$$\frac{\partial v}{\partial \mu} > 0, \ \frac{\partial v}{\partial \sigma} < 0$$

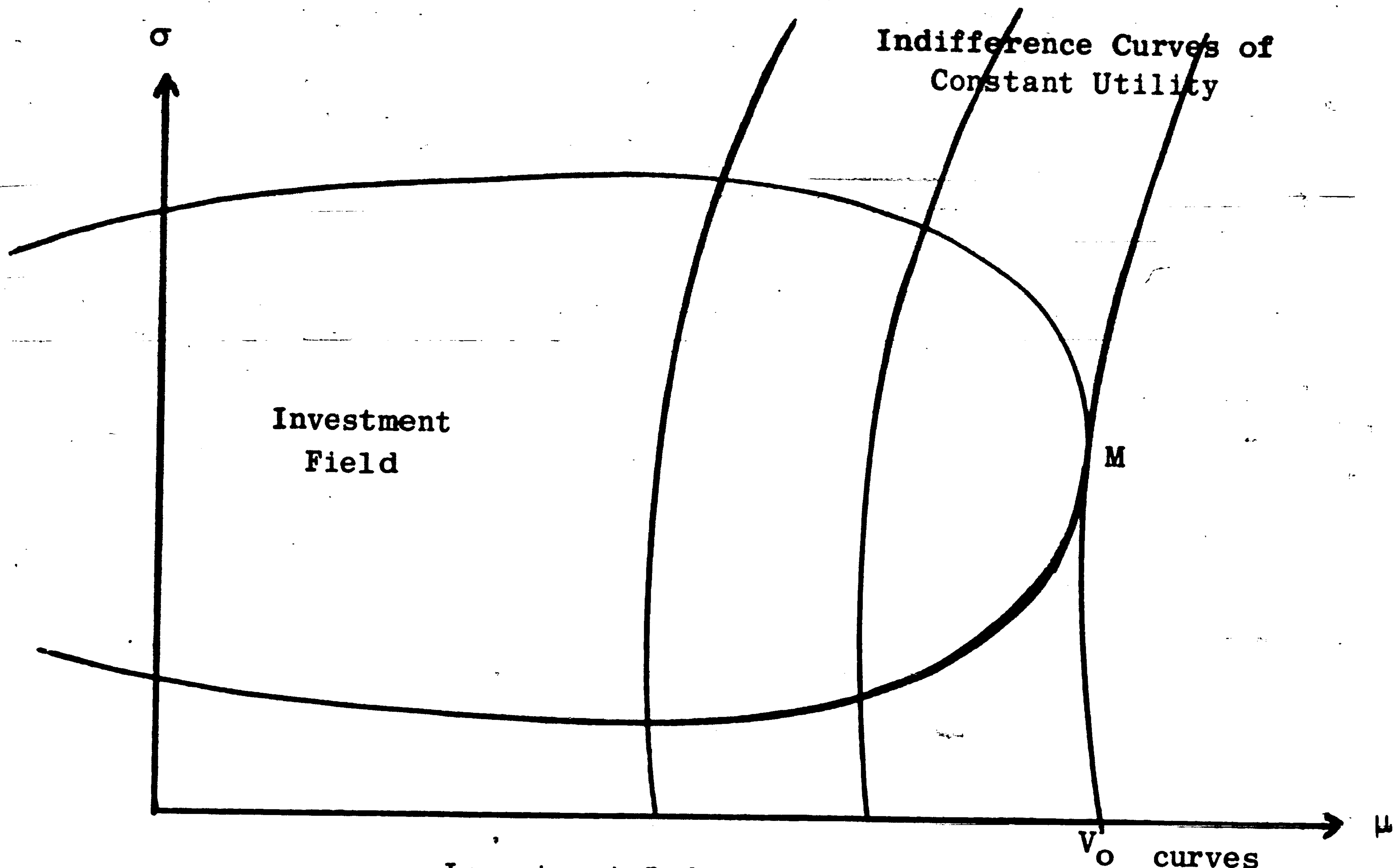
and

$$\frac{d\sigma}{d\mu} = - \frac{\frac{\partial v}{\partial \mu}}{\frac{\partial v}{\partial \sigma}} > 0$$

Also, if risk disappears:

$$v = f(\mu,0) = \mu$$

The choice of the proper alternative involves selecting from the graph, Figure 11, an investment whose expected value and risk are equivalent (M) to the investor, to a certain receipt of v_o dollars.



Investment Indifference Curves

FIGURE 11

In fact, "this solution is, in most respect, identical to the classical equilibrium of consumer choice---It envisions the maximization of an objective function subject to the constraint of available investment opportunities. Geometrically, it consists of the usual tangency between an indifference curve and the border",[1] of the investment field.

Little development of the theory has been done beyond this point. For example, "only few economists have heretofore attempted to specify even the form

$$v = f(\mu, \sigma)$$

which a certainty equivalence relationship is assumed to take, and none has attempted to go so far as to evaluate the parameters that define

[1] (11, p. 14).

such a function once its general shape has been decided upon", (11, p. 15).

Incorporation of risk and uncertainty into a workable utility theory has been mostly the work of Von Neumann and Morgenstern (34), Friedman and Savage,[1] and Markowitz.[2]

Von Neumann and Morgenstern implied that individual utility can be measured. They made the following assumptions:

a. An individual can give a preference order for all alternatives; i.e., an order scale can be established.

b. An individual can also express preferences for combinations of alternatives and stated probabilities; i.e., an interval scale can be incorporated in which it is possible to order probability combinations of states.

Many economists found it difficult to accept part "b", perhaps because the proof of the theorem was so difficult and because the preceding generation of economists had waged a fierce battle over the measurability of utility.

To examine these implications that risky propositions can be ordered in desirability, that expected utility is behaviorally meaningful, and that choices among risky alternatives can be made in such a way that they maximize expected utility, it is necessary to investigate the concept of lotteries.

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[1] Refer to their article, the Utility Analysis of Choices Involving Risk, Journal of Political Economy, Vol. 56, 1948.

[2] Refer to his article, the Utility of Wealth, Journal of Political Economy, Vol. 60, 1952.

If a firm prefers alternative A to alternative B and alternative B to alternative C, then, there exists for the firm a lottery in which A and C is preferred to the certainty of getting B. If, for example, the chance of A occurring is 0.95 in this lottery and the chance of C occurring is 0.05, it appears logical to assume that the firm would choose the lottery rather than the certainty B. But, if the probabilities for A and C were reversed, then the firm would choose the certainty of B. Thus, there must exist some probability in between these extremes where the firm will be indifferent to B, for certain, or the lottery of A and C in which the probability of A would be p and that of C, 1-p. As p for A increases from 0 to 1 the preference for the certainty option gives way to the preference for the lottery option.

The axioms which underline the utility theory usage above are:

a. The decision maker can arrange a complete and transitive ranking of the alternatives; i.e., outcomes.

b. Any prospect of gamble involving equally desirable outcomes is just as desirable as either outcome by itself.

c. If outcomes A, B, C are ranked so that A is preferred to B, B is preferred to C, and A is preferred to C, then there exists a gamble involving A and C which is just as desirable as B.

d. If A and B are equally desirable, then the gamble $pA + (1-p)C$ is just as desirable as the gamble $pB + (1-p)C$.

One can prove that a value scale does indeed exist if these axioms hold. But, ~~can we~~ be sure that the axioms do hold in the firm and that the decision maker can properly reply to such question as might be posed. "There is not yet sufficient evidence to argue conclusively one way or the other on these questions." (21, p. 157). Actually, the six axioms listed in Appendix 1 are the foundation upon which modern utility theory has been built. These axioms, although slightly modified to reflect an investment flavor, are similar to those developed by Von Neumann and Morgenstern (34). Detailed interpretations of these axioms can be found in Luce's and Raiffa's book (16, Chapt. 2) or in Prof. Ackoff's book (1, Chapt. 3).

Many examples of behavior can be generated which violate the axioms of Von Neumann - Morgenstern,[1] especially if the amounts of money involved are very large, or when, the probabilities involved are extremely small. But, on a general evaluation basis, it may be correct to state that their theory has "considerable predictive power".[2] This was confirmed by the famous controlled experiments made in 1950 by Mosteller and Nogee (71). They studied a group of Harvard undergraduates and some members of the Massachusetts National Guard and attempted to:

a. Construct a utility curve for each subject from the behavior of a game of dice.

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1 For example, refer to Strotz, R. H., Cardinal Utility, American Economics Review Supplement, Vol. 43, 1953, pp. 384-405.

2 Expression used by Prof. Karl Borch in discussing Von Neumann-Morgenstern's theory, in Economics of Uncertainty, Working Paper 79, Un. of Calif., 1965, Chapter VI, p. 11.

b. Make predictions from the utility curves about future individual behavior toward other more complicated risks, and to test these predictions.

It turned out that the Harvard undergraduates had a curve of diminishing marginal utilities,[1] while the National Guardsmen had increasing marginal utilities. In addition, these curves were fairly accurate in predicting future behavior toward riskier situations.

Friedman and Savage, and Markowitz have been primarily concerned with the effects any change in wealth or income would have on a firm's or individual's current income utility curve. Friedman and Savage's utility curve for money is shown in Figure 12.

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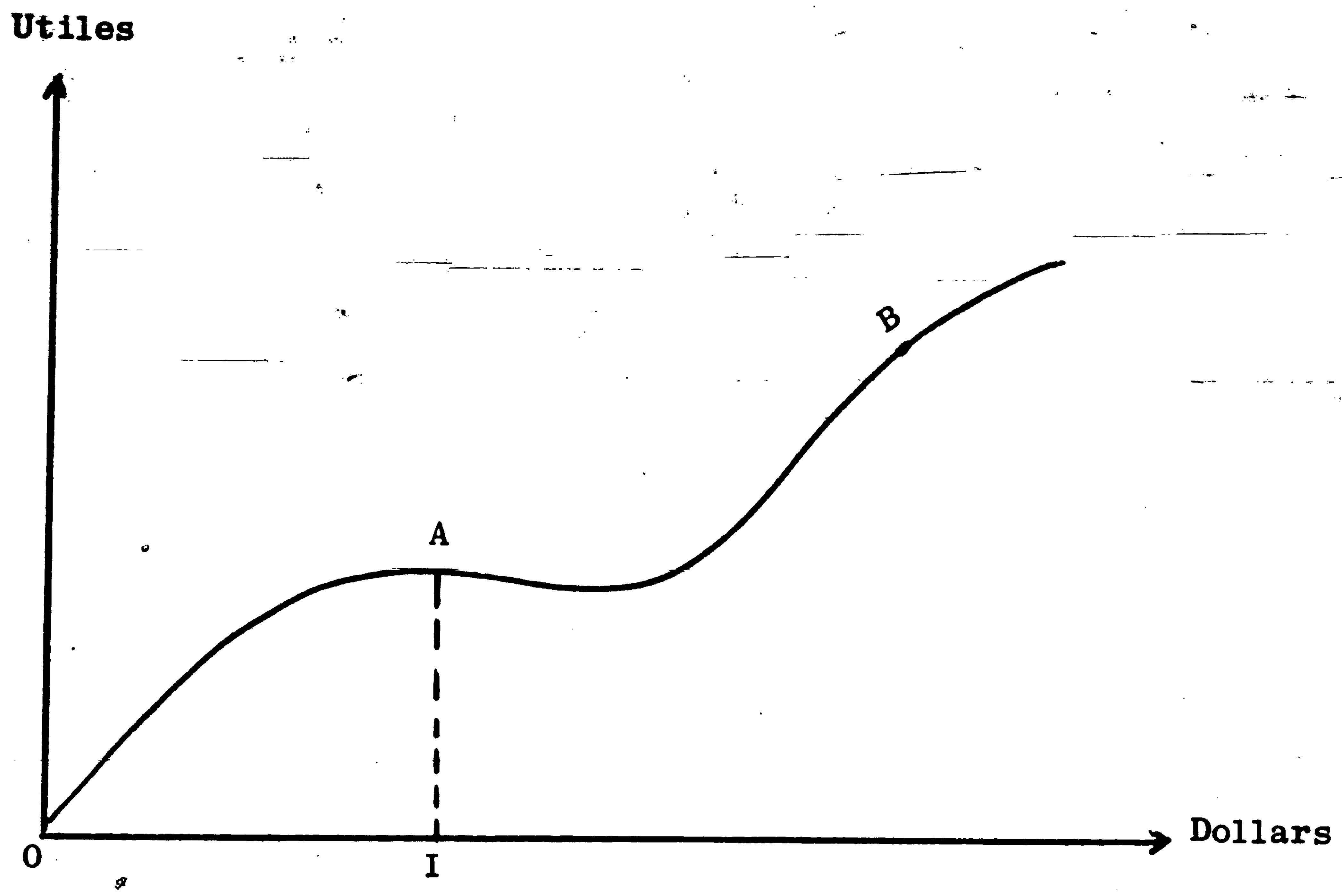
[1] Marginal utility is the rate of change in total utility per unit change in quantity and is not the utility of a marginal unit.



I = Current Income or Wealth

OA = Decreasing Marginal Utility (consistent with avoidance of risks)

AB = Increasing Marginal Utility (favors risk undertaking)

FIGURE 12

Hypothetical Utility Curve for Money, Proposed by Friedman and Savage

Actually they developed their curve to explain why the same person will buy insurance and will gamble. Friedman's and Savage's curve is dependent upon levels of income rather than upon changes in income and it is presumed that individuals choose as if they were moving along that curve. In other words, the utility curve stays put and the individual moves up and down as wealth or income changes.

Markowtiz suggested that the origin of a person's or firm's utility curve for money be taken as his customary financial status and that on both sides of the origin the curve be assumed first concave and then convex. If a firm's customary state of wealth changes, then the shape of his utility curve will thus remain generally the same with respect to where he now is, and so his risk taking behavior will remain about the same instead of changing with every change of wealth as in the Friedman-Savage proposal.

It is possible to derive demand curves from utility functions. This may be important to the decision-maker when considering his own demands or from the viewpoint of considering his customer's utility/demand interactions. To illustrate how such a demand curve may be derived, consider the following utility function:

$U = \log x + \log y$. Assume that P_x, P_y, and I are given where:

x = number of units of commodity x $\qquad x = 1,2,3,\ldots$
y = number of units of commodity y $\qquad y = 1,2,3,\ldots$

P_x = a set price for commodity x

P_y = a set price for commodity y

I = money income or wealth

It is easy to show[1] that the condition for maximization is

---

[1] Differentiate the following with respect to X, Y, λ:

$$U(X,Y) + \lambda(XP_x+YP_x-I)$$

to get

$$U_x + \lambda P_x = 0$$
$$U_y + \lambda P_y = 0$$
$$XP_x + YP_y - I = 0$$

$$\frac{U_x}{P_x} = \frac{U_y}{P_y}$$

where U_x = partial derivative of U with respect to x.

U_y = partial derivative of U with respect to y.

This in effect means that the "marginal utility per penny's worth of commodity x must equal that of commodity y", (12, p. 40).

Because:

$$U_x = 1/x$$

$$U_y = 1/y$$

Then

$$\frac{1}{xP_x} = \frac{1}{yP_y}$$

Therefore,

$$xP_x = yP_y$$

But, the money constraint is:

$$xP_x + yP_y = I$$

thus,

$$2xP_x = I$$

$$x = \frac{I}{2P_x}$$

and this is the demand curve.

Prof. M. Friedman in (12) presents a remarkable conclusion by examining the following three different utility functions:

TABLE VI

Utility Function and Derived Demand Functions

Utility Function	Demand Function	Characteristics
$U=\log x + \log y$	$x = \frac{I}{2P_x}$	Marginal Utility of x and y are independent. Demand Curve is equilateral hyperbola.
$U = xy$	$x = \frac{I}{2P_x}$	Marginal Utility is constant and there is dependence. Marginal Utility of x=y; i.e., ($U_x=y$) and that of y=x; i.e., ($U_y=x$)
$U = x^2y^2$	$x = \frac{I}{2P_x}$	Marginal Utility for both x and y are increasing and there is dependence. Marginal Utility of $y=2x^2y$ and Marginal Utility of $x = 2xy^2$

This unexpected conclusion lies in the fact that "in each case we end up with the same demand function. This seeming paradox can be stated in another manner. We notice that people spend one half of their income on commodity x, which is the case when the demand function is $x = \frac{I}{2P_x}$. Yet there are three different utility functions which rationalize this observed phenomenon", (12, p. 42).

In summary, whenever a decision-maker assigns utilities to events or proposals, "he is really saying that in his judgment his long-run expected profit is increased by playing it safe until he has built up greater financial strength and therefore he will choose an act whose immediate expected profit is less than that of some other, riskier act", (26, p. 48).

CHAPTER VI GAME THEORY AND CAPITAL INVESTMENT

Game theory[1] may be viewed as a mathematical method for analyzing problems that arise out of conflict. Since most investment decision problems can be regarded as a conflict (between objectives), we may inquire whether management is making use of this tool in evaluating alternatives.

Consider the following example due to Edward G. Bennion (43):

Suppose we know that:

a. The most probable forecast is for a recession.

b. In recession, investment in plant will yield 1% as compared with a 4% yield for securities.

c. In prosperity, plant will yield 17% while securities will yield 5%.

Now if we form a 2 X 2 game matrix we have:

Management Investment Alternatives	Cycle-Phase Alternatives (Nature's): Recession	Prosperity
Securities	4%	5%
Plant	1%	17%

In order to know how probable the "most probable" forecast is, we need, usually from the economists or statisticians, a probability coefficient, i.e., a forecast probability, for each cycle phase. Let us

1 The pioneering work in game theory was written by John Von Neumann and Oskar Morgenstern in 1944 (34). Appendix 3, "Game Theory - A Brief Review", surveys the basic theory. It should be noted that the game theory formulation which deals with decision making under certainty is in reality linear programming and as such is not treated in this thesis.

suppose that the forecasters think the chances of recession are 6 out of 10. (Thus, the recession probability coefficient = .6 and the prosperity probability coefficient = .4.) At this point, we must also calculate the indifference probabilities which are a unique set of probabilities describing the situation which exists when each course of action is equally attractive. The calculations involved are as follows:

Let R = recession probability coefficient

P = prosperity probability coefficient

Then, from the matrix it is known that:

(a) $4R + 5P$ = return on securities

(b) $1R + 17P$ = return on plant

Accordingly, it is seen that the return on securities will be the same as the return on plant if:

(c) $4R + 5P = 1R + 17P$

Solving the last equation for R in terms of P gives:

(d) $R = 4P$

But, since the sum of all probabilities $(R + P)$ must equal 1.0, we obtain

(e) $1 - P = 4P$

or, the indifference probabilities are

(f) $P = 0.2$

$R = 0.8$

To sum up in matrix form:

		Cycle-Phase Alternatives (Nature's) Recession	Prosperity
Management Investment Alternatives	Securities	4%	5%
	Plant	1%	17%
Indifference Probabilities		R = 0.8	P = 0.2
Forecasted Probabilities		$\hat{R}$ = 0.6	$\hat{P}$ = 0.4

By comparing the forecasted probabilities[1] with the indifference probabilities, the best alternative is noted; i.e., if the indifference probability for a given alternative is greater than its forecasted probability, the decision maker would not select this alternative. The rationale for such action lies in the fact that it requires a probability greater than the forecasted one to make this alternative's attractiveness equal to the other possible choices. Returning to the example, if the probabilities of a recession and prosperity are 0.8 and 0.2 respectively, then the chances are the firm will be just as well off investing in securities as in plant and vice versa. Accordingly, the decision maker knows, based on these probabilities, "that he is not making an avoidable mistake by playing for high stakes and building a plant", (43).

What has this decision maker gained by the use of the game matrix?

a. He has become aware that the best paying investment

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1 These probabilities can be viewed as the opposition's or nature's probabilities and represent the probabilities of nature to hold the firm's gains down to a minimum.

alternative in the most probable situation is not necessarily the one that should be selected.

b. Using indifference probabilities, he can compute the margin of error which may result in an erroneous decision.

This technique for a zero-sum game can easily be extended to a 3 X 3 or larger matrix.[1]

The capital budget decision maker attempts to maximize or minimize some given index as a preliminary to choosing the proper alternative. The choice of this index is crucial. Usually in capital budgeting, the dollars of profit or loss, or the per cent of return on investment, etc., is the appropriate index. "The essence of the problem is: how should the subject select an index function so that his choice reduces to finding the alternative with the maximum index." (16, p. 15). Manipulation of these indices must be in accordance with several assumptions or axioms of modern utility theory.[2]

It is not always possible for a firm to make a model of its preferences in any given capital situation or to be able to establish a set of utility assignments to the various capital budgeting outcomes. These factors can offer limitations to the employment of game theory. Because different firms at different times have different indices,[3] no

[1] Edward G. Bennion includes in his articles an actual case of an integrated petroleum company using a 3 X 3 matrix (43, p. 118). This case is interestingly presented and shows how game theory alters the conventional decision maker's selection of the "proper" alternative.

[2] Appendix 1, Utility Theory Axioms From the Viewpoint of Capital Budgeting, lists these axioms.

[3] For example, if a firm must select the alternative to match a competitor's move to advanced technology, anticipated profit from the capital budget may be a secondary consideration.

common utility index can be established in a general treatment of game theory to capital budgeting. This has handicapped the development of the game theory approach. Limitations, other than this, which have arrested the application of game theory are:

a. The lack of experience which the decision makers have with game theory.

b. Little has been written on the subject of game theory and investments. Recent books on decision making make no mention of game theory. These books seem to compromise by including tree techniques and lottery methods.[1]

c. The definition of rational behavior on part of the decision maker has been "an incessant cause of controversy between the followers and disbelievers of the theory. As is so often the case, a good part of the dispute has been terminological rather than substantive". (81, p. 371).

d. When we enter n-person games, a "new realm of issues"[2] appear. "In spite of the obvious importance of n-person games, relatively little research has been done in this area, and applications are few and far between." (81, p. 385). In fact, when Luce and Raffia deal with n-person games, they devote one fourth of their book to the subject; however, they "lack a persuasive theory

[1] For example, (a) The textbook which has only been introduced provisionally in 1965--Pratt, John W., Howard Raiffa, Robert Schlaifer, Introduction to Statistical Decision Theory, McGraw-Hill, Inc., (22), (b) Schlaifer, Robert, Probability and Statistics for Business Decisions, McGraw-Hill, 1959, (26).

[2] (81, p. 384).

for n-person games", (81, p. 384).

e. In addition, there are some game theorists who disagree on what an optimal strategy and rational action are in the non-zero game.

f. A review of Appendices 2 and 3 would reveal that:

1) Mixed strategies may fail because of a technical problem -- it is impossible to achieve complete randomness. This bias which then must exist can be taken advantage of by an intelligent opponent.

2) Minimax criterion may fail because complete ignorance of a competitor's relative probabilities of action is impossible. If the opponent's alternative can be defined, then the player must know something about them.

Because of such objections to game theory, Lawrence Friedman in (54), attempts to develop a gaming model which:

1. Has one and only one optimal course of action.
2. Has one and only one objective function: the maximization of expected utility.

But on the other side of the coin, what may the decision maker hope to gain by employing game theory:

a. The decision maker will utilize all forecasts. In the conventional approach to selecting an alternative, the decision maker may only select the most likely forecast. The other forecasts may not really concern him. In game theory, all the forecasts are employed to assist the decision maker. This apparently makes for more accurate output.

b. Margins of errors can be calculated by the use of indifference probabilities.

c. His evaluation of rating alternatives can be assisted by this scientific technique. By the use of game theory, he is forced to think seriously of his utility preferences, and this may cause him to be more encompassing when reviewing pertinent factors.

d. Once game theory is employed consistently, more uses can be developed for the theory. For example:

   1) The use of game theory to generate priority ratings for alternatives. This proposal has been offered by Edward G. Bennion (43, p. 207).

   2) An analysis of problem areas such as input, by working backwards, may be possible. Because game theory is assumed correct, and if the results are wrong, the inputs must be at fault. A backward search could point out where the input was in error, and it could also point out how much the probabilities assigned to the forecasts were off.

In summary, it can be stated that game theory does have some potential in investment application; but, little has been done to exploit and develop this potential. Several theoretical limitations have been responsible, in part, for this lack of general usage.[1]

Although game theory has been treated as the proverbial stepchild in relation to investments, it has enjoyed a better treatment and development in the general field of economics.[2] Economic applications considers:

a. The possibilities of communication between opponents.

b. The degree to which each player trusts the other.

c. The magnitude of payoffs.

d. The number of times the game will be played.

e. The application of Nash's criteria for a unique solution to the two person bargaining problem.

Classical examples which can be used to illustrate these above points are "The Prisoner's Dilemma", (16, p. 94), or "The Battle of the Sexes", (16, p. 90).

At this point, Prof. Wagner's comments about the future of game theory are pertinent:

"Although Luce and Raiffa do not devote a particular section to the likely new developments and future influence of game theory on the social sciences, certain indications seem clear. The Bernoulli

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1 An interesting but simple example of the use of game theory by management to help resolve problems other than investment is given by the authors of (5) in Chapter 8. They attempt to show how a union/company dispute may be resolved by game theory and utility theory.

2 An excellent reference is Prof. Karl Borch's Working Paper #83, "The Economics of Uncertainty", Chapter X, University of California, June 1965.

utility function[1] will continue to provide the mainstay for the analysis of decision problems in stochastic situations. Although the minimax strategy may not be offered as the 'optimal' rule for the selection of a strategy in the face of uncertainty,[2] it will remain a procedure worthy of serious consideration in such circumstances. Development in n-person theory are likely to be of a dynamic and behavioristic nature, perhaps embodying Marchak's notion of a 'theory of teams'. Consequently, there will be a tendancy to drop the normal form abstraction of a game." (81, pp. 386-7).

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[1] A Bernoulli utility function for uncertainty is defined as follows: Given the utilities for a set of nonrandom events, the utility for a lottery with these outcomes, each occurring according to a given probability distribution, is numerically equal to the expected value of the utilities for the certain events. Therefore, Bernoulli utility implies a linearly additive function of the component utilities for each possible lottery prize.

[2] Refer to Appendix 2, "Six Rules Necessary for the Employment of the Minimax Strategy".

CHAPTER VII SUMMARY AND CONCLUSIONS

In the last few decades, numerous investigators have attempted to develop a conceptual (and/or operational) framework which could be of assistance in capital investment decision making. This investigation has in turn attempted to present these contributions in light of their strength, weaknesses, inconsistencies, and areas of conflict.

Accordingly, in the preceding pages it was shown that:

1.0 Capital investment decision making occupies a central roll in business. This is so because:

(i) Huge sums of money are involved.

(ii) Both the short and long range company profitability, competitive position, and perhaps its very survival, are affected by these decisions.

(iii) Capital expenditures, from an aggregate and cyclical point of view, have a great deal to do with the character of the economy as a whole. Reflectively, the company itself is then affected.

(iv) Investment decisions usually have long lasting effects. Mistakes in investment decisions cannot be worked off in a short period of time. If necessary, the dollar penalty for reversing a decision can be very high.

(v) These decisions are management's tools of strategy; i.e., the means by which the "direction" of a firm are controlled.

(vi) All levels of the firm's personnel from stock holder to laborer are effected by the fruits or losses of capital investments.

2.0 The major difficulties faced by the decision maker are:

(i) The risk and uncertainty of the future together with the concomitant assumptions which must be generated.

Because capital investments deal with the future, risk and uncertainty are imbedded in each of the following: the forecasts of expected costs, revenues, profitability, utility, and environmental conditions; e.g., per cent of market. Short-range forecasts are usually reasonably accurate because there is a carry-over of present knowledge into the immediate future. At least the reliability of the data is greater, the closer into the future we do our planning. But, for long-range planning, the decision maker is not so fortunate. The present inability to forecast accurately is due to many factors; chief among these is a basic lack of understanding the casual relationships that exist among the variables involved. This state of affairs is then aggravated by the rapid changes in technology and science. It appears then that the decision maker is almost cast in the role of Sisyphus, because, even as he begins to understand the relationships that exist among the variables, technological advances change these relationships. By the time the decision maker begins to comprehend what is happening and why, the casual relationships have changed. It then comes to pass that he understands a problem that is no longer. Thus, although, the decision maker will never operate in a future under conditions of certainty, he requires more reliable data, better estimates and forecast in order that he may, among other things, push the immediate future time period within which he is fairly confident, a little further out.

(ii) Questionable reliability of the estimates and forecasts used in decision making

Emphasis has only recently been placed on this important need. As a result, error analysis techniques, risk assessment methods such as W. L. Gore's procedure, sensitivity analysis, sampling from subjective probability distributions of concerned variables, and the use of confidence limits have come into being. At present, these are the "better ways" of securing more reliable estimates and forecasts. But, more research is still needed in this area.

(iii) Goals which are not or cannot be clearly defined.

Good planning requires:

(a) All goals, especially long-term goals, be defined as clearly and precisely as possible. Because the achievement of a goal involves the interaction and dependence of many factors, its realization exists in a conditional environment. Accordingly, a good definition of a goal should involve the probability distribution of the pertinent factors, such as anticipated demand, expected sales, expected size of market, anticipated size of capital expenditures, expected growth of the industry, expected volume of new military business, etc. Then, various convolutions of these factor distributions should be made to show the probability or odds of actually reaching the goal, the influence each factor exerts through its sensitivity or leverage effect and its relative degree of uncertainty, and the limits for each factor within which the goal can be realized. Purely verbal descriptions are the poorest method for defining a capital investment goal.

(b) Avoidance of unrealistic plans. Actually, this usually follows directly from "a" above, because, if goals are not clear, it is possible to generate impossible or unrealistic goals.

(c) A differentiation between the means of achieving a goal and the goal itself. If goals are poorly defined, it is possible to mistake the method and means to achieve them, which are usually given in detail, as the goals themselves. This is placing the emphasis on how to do something rather than on what to do.

3.0 The major stumbling blocks in evaluating a capital investment proposal are:

(i) The lack of widely accepted criteria. This is true whether the decision maker is concerned only with the methods of measuring the profitability of a proposal, the cost of the various types of funds, or the effects of capital structure on the cost of capital.

(ii) The difficulty in accumulating experimental data of a comparable nature; i.e., almost all investment projects are unique. This prevents an objective statistical approach such as the use of the Central Limit Theorem.

(iii) The lack of a discipline which specifically may serve the decision maker. By contrast, the physician has his allied field of medicine and the lawyer his study of law.

4.0 The more generally accepted methods for evaluating investment alternatives are:

(i) PAYBACK

(ii) PRESENT VALUE

(iii) DISCOUNTED CASH FLOW

(iv) SUBJECTIVE

(v) MAPI

(vi) ACCOUNTING

(vii) NECESSITY/POSTPONABILITY

(viii) DECISION TREE

4.1 These methods vary in their usefulness, mainly depending on the characteristics of the investment alternatives. This can be best summarized by considering:

(a) A "sensitivity" scale.

(b) A "particular to general" scale.

(c) A "degree of complexity" scale.

The sensitivity scale recommends a given technique or techniques if the decision maker is primarily concerned with one factor of the project, while all other factors remain unchanged. Table VIII is such a scale representation.

The "particular to general" scale demonstrates the applicability of the various evaluation methods. Such a scale may be illustrated as follows:

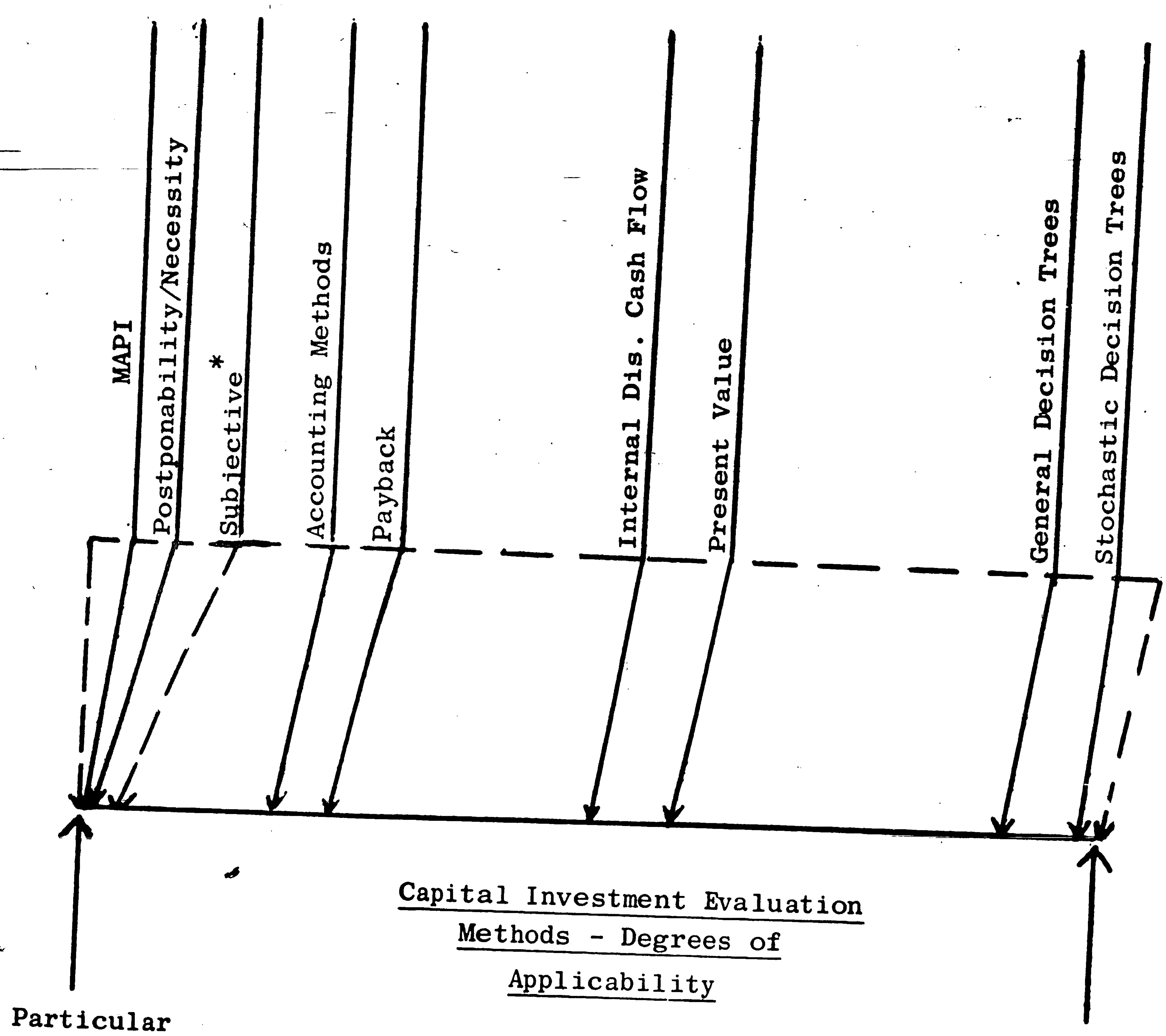
TABLE VII

Recommended Computational Techniques When The Factor That Is Of Chief Concern To The Investor Is Given

TABLE

Factor	Present Value	Internal Rate of Return	Payback	Sub-jective	Decision Trees	Acct. Methods	MAPI
Risk			**		*		
Liquidity			*				
Profit-ability	**	***			*		
Fast Screening			*			**	
Legal/ Necessity	**			*			
Interest Rate						*	
Timing	*	**					
Accept/ Reject	*	**			**		
Ranking	*	**					
Equipment Replacement	*						*

Legend: * is preferred to **; ** is preferred to ***.



Capital Investment Evaluation Methods - Degrees of Applicability

FIGURE 13

* When used as the sole technique of evaluation, subjective evaluation belongs at the extreme left of the scale. However, subjective judgment and experience is used and should be used together with other criteria in making a final decision. These criteria are only aids to the decision maker and as such should never be used alone to judge the worth of a proposal.

The relative complexities in applying these capital investment methods are as follows:

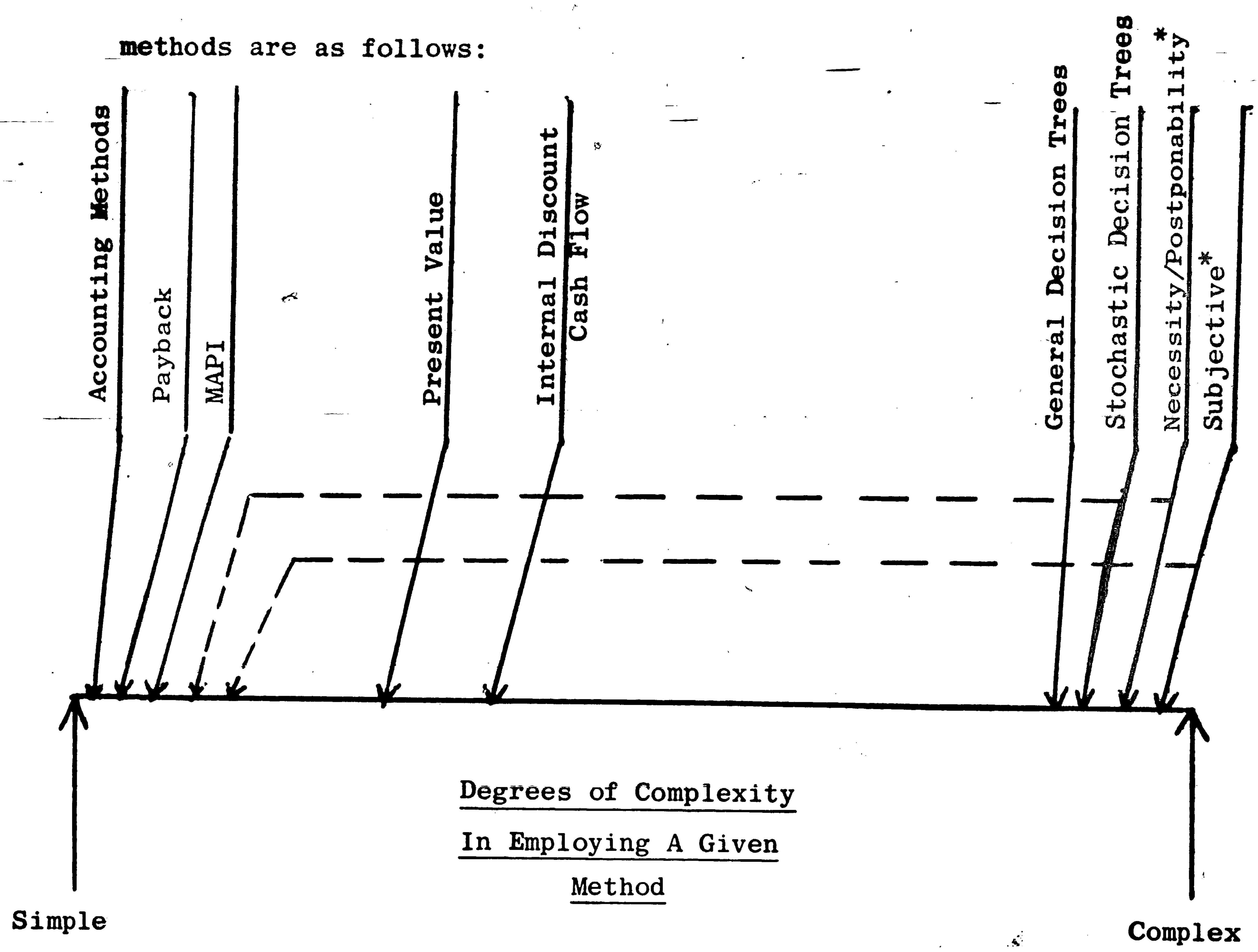


FIGURE 14

* Necessity/Postponability and Subjective Methods can be extremely difficult to apply if a full understanding of the variables involved and their effects are not understood or known. On the other hand, if the proposal is relatively simple with clear-cut objectives, etc., then these methods may be quite easy to apply.

4.2 The three most widely accepted methods for measuring the "profitability" of capital investment proposals are:

(a) Discounted Cash Flow

(b) Present Value

(c) Payback

The fundamental properties of these three methods can be summarized as follows:

(a) Discounted Cash Flow Method

Developed in 1951 by Dr. Joel Dean, this method was justly regarded as a great step forward. Particularly effective in evaluating independent projects, the internal yield method will not work properly where two or more projects are dependent. To apply this procedure, the firm must compute the internal rate of return for each investment proposal and then rank these proposals in decreasing order. The marginal cost of capital determines the cut-off or minimum acceptable rate of return which can then be used for selecting projects.

An additional requirement is that an unlimited or unconstrained capital budget fund exist in all time periods. The average firm does not have these unlimited funds. Generally, there are more proposals than money available and it is not so simple a question as deciding which project's rate of return exceeds the minimum acceptable rate. The basic problem involves the selection of the best combination of proposals within the constraints of budget funding.

Even if it were correct to employ this method when funding is limited, it would be difficult to use it since yields are <u>not</u> <u>additive</u>. It is possible to add profits and present values, but one can not add internal yields in the same context, to determine an "optimum" combination of projects.

Finally, this method seems to be applicable in evaluating conventional proposals; (i.e., streams of returns are not affected by alternations in sign). If this condition of "conventionality" is not satisfied, it is possible to have no return or multiple returns. Such a state of being does not reflect the desirability of a proposal. This problem, however, has been resolved by special sophisticated treatments by Lorie and Savage, Merrett and Sykes, and a few other investigators. The average decision maker is usually ignorant of this fact.

When outlays take place over several periods, the discounted cash flow changes into an average annual rate of return which indicates the average rate of yield over the entire life of the project. For example, an outlay of $20 now, followed by a subsequent outlay of $10 a year hence and a return of $35 two years hence, can be said to have an internal rate of return of 10%. The correctness of this rate can be tested by assuming that one invests corresponding amounts in a bank at 10%. But, the process of computing the internal rate of return is to discount the expected cash flows in order to find that rate at which net present value is zero. Doing this seems to suggest that the outlay of $20 actually earns 10% between its investment date and one year later, whereas, in fact it yields nothing during this period.

Overall, this method is based on accounting criteria rather than economic theory concepts. In particular the notion of utility is not and cannot be considered in conjunction with the internal rate of return. An application of utility involves the mean and variance of a variable. The present value method can supply a meaningful mean and variance; since one can interpret present value as the "expected" amount of a discounted cash flow process which the firm anticipates receiving. The variance of this present value can be related to the risk involved. But, it is meaningless to "speak" about the variance or sigma of an internal rate of return.

To illustrate the above, assume that the utility function can be reasonably approximated by a quadratic: $U = a+bx+cx^2$ where a,b,c are constants and x is the expected cash return. Then,

$E(U) = a+bE(x) + cE(x^2)$. But, since $E(x^2) = \sigma^2(x) + [E(x)]^2$, it is readily seen that it is only necessary to calculate the expected value and variance of the payoffs. Actually, it is possible to view the variance as a measure of risk appropriate to a quadratic utility function.

In an environment ideal for the discounted cash flow method, the ranking and accept/reject results obtained with this technique will always be similar to those of the present value method. Thus, it matters not which method is used. But, such assumptions are usually unrealistic because investment proposals usually interact and there is a limit to available funds.

If one wants to employ the internal discounted cash flow method on mutually exclusive proposals, it is necessary to compute the yield on the incremental cash inflow in order to determine which of a pair of such investments is preferable. If more than two mutually exclusive alternatives are being considered, it means that the return on the incremental cash flow of the first two alternatives must be calculated and then the better of this pair decided. The "winner" of the first round is then paired with the third alternative and its incremental benefit return is calculated, etc. Any ranking by this technique will usually differ from a comparable present-value ranking. This is due to the different inherent reinvestment rates of both methods. As a rule, the present value ranking is more reliable and accurate. Proposals on how to overcome these differences; e.g. use of average rates of return, have been examined in detail in this thesis.

(b) Present Value Method

Some of the advantages of this method are:

1. Present values, unlike internal yields, are additive; and hence, can readily fit into a structure of selecting a composite of the "best" proposals within budget constraints.

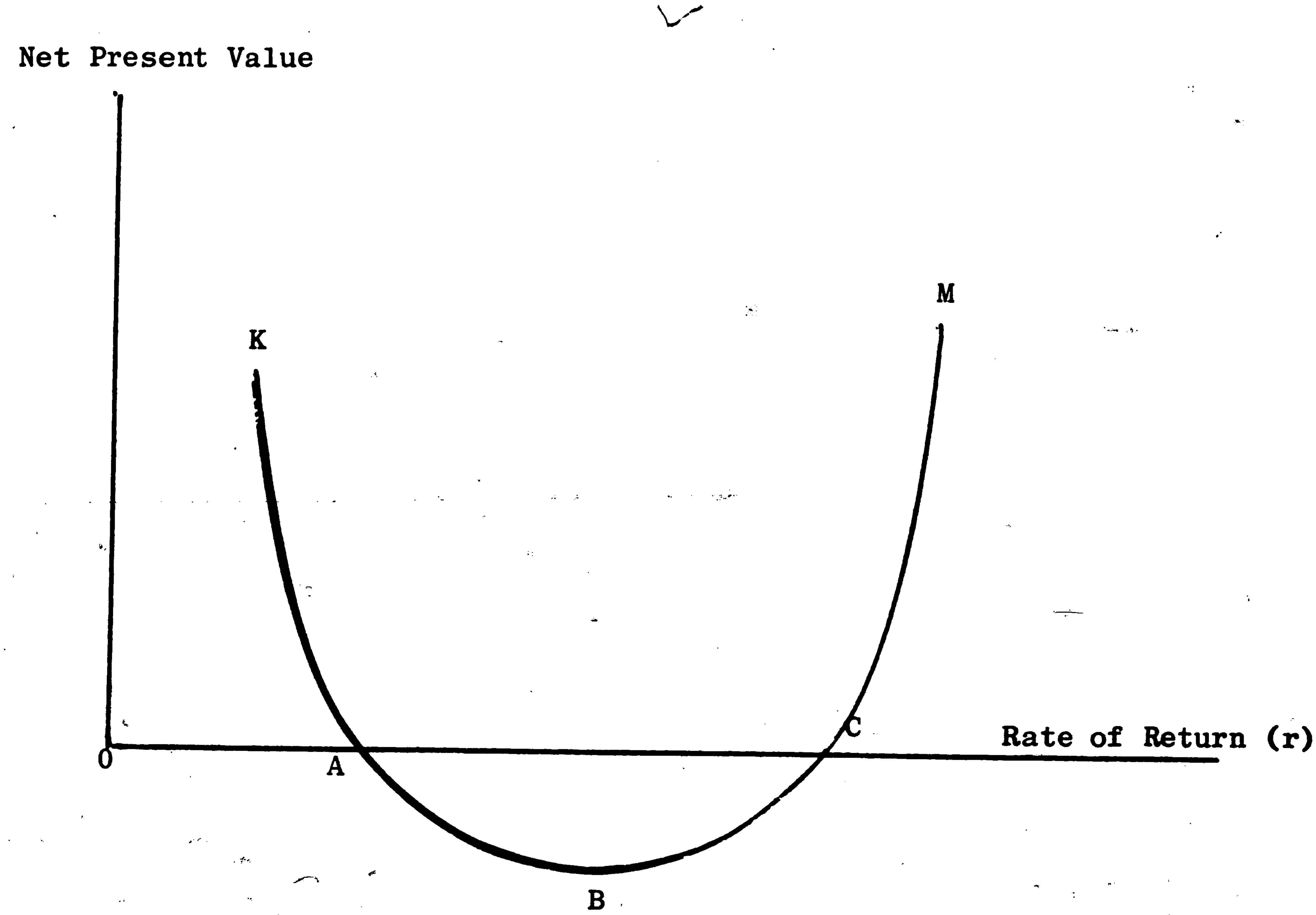
2. Present value can readily be incorporated into a utility theory concept. This has been detailed in "a" above.

3. Present value concept can be enlarged and built upon and offers a potential for future investigators. Decision trees, standard and stochastic, incorporate present values in their mechanics. A marriage between the present value method, risk evaluation, and utility into an operational mathematical model would be most welcome and useful.

4. It gives reliable answers in the type of "environments" that the internal yield method may fail;

e.g., dependent proposals and non-conventional proposals.

However, unless the decision makers appreciate what is happening, the present value method can, under certain cases, appear illogical and not intuitively correct. For example, as the interest or discount rate increases, one would expect the present value to continuously decrease. In other words, as r increases, $(1+r)^{-1}$ becomes smaller. But, whenever multiple rates of return exist, the present value of the proposal may first decrease and then actually increase as the rate of return increases. Graphically, this can be represented as:



As r increases, present value decreases from K to A, and from A to B. Then, as r continues to increase, the present value continues to increase from B to C to M. This just seems to defy one's intuition. What is actually happening, is that the more distant outlays of the non-conventional proposal in comparison with present outlays are playing a less influential role.

5. Negative net present values may be acceptable to a decision maker.

Because proposals generally interact and funds are limited, a negative net present value may be acceptable. For example, assume cash flows for a given project are as follows:

t_o : \$ -10
t_1 : \$+100
t_2-t_4: 0
t_5 : \$-300

The investor would be willing to accept the "losing" investment in order to obtain \$100 in the near future against his immediate outlay of \$10. This \$100, (actually \$90 which he did not have earlier), would enable the decision maker to reinvest in other proposals which are deemed profitable. Thus, to an investor, short of funds, a negative value proposal may be accepted.

The disadvantages of this method are:

1) Risks and uncertainty of all proposals are considered non-existant or equal.

2) The weighting factor, exponentially decreasing with time, which is almost invariably used for discounting, is completely arbitrary.

Money invested in a savings bank account will grow according to the exponential rule of Pe^{rt}, (where r is the rate of interest, t is the time period involved, and P is the sum invested at present), because it has been so decreed. But, money invested in government savings bonds or industrial equities do not, in general, behave in accordance with this formula. Perhaps the fact that certain investments use a different growth formula is some evidence that the exponential one is not necessarily the best. No doubt, this exponential formula is being used, not because it is the best, but because it is the simplest.

(c) Payback

In an environment where there are no restrictions on budgeting, payback method usually serves no useful purpose. The present value or internal rate of return would be more appropriate because no single proposal could "hurt" the firm and the involved risks are usually minimal.

This method, however, becomes a very useful index when funds are very limited, the future very uncertain, and when the returns are needed in the relatively near future for reinvestment.

Actually, in some sense, it is possible to view payback as a conservative method. It has as its "objective function" not the maximization of profit but the maximization of cash in the near future.

It is important to realize that payback measures only the risk associated with time since it assumes that a project continues uniformly with unchanged profits for a certain period of time and then suddenly ceases to be and ceases to have any value. It does not measure the risks associated with normal business dynamics; e.g., sales may decrease, costs may increase, taxes and inflation may increase. Even if payback is used for time risk, it is still imperfect, since it does not make allowances for the time costs of money nor the amount of the initial capital investment recovered. Its other shortcomings have been detailed in this thesis.

4.3 A failing of all the methods examined above is the fact that the risks and uncertainties are ignored or are assumed equal.

In an attempt to incorporate risk into their planning, many decision makers have incorrectly adjusted their cost of capital or else have turned to the payback method.

Much of the present day research has been concerned with incorporating risk into an operational model for measuring profitability. Accordingly, recent emphasis has been given to "decision tree" analysis and "stochastic trees" in particular.

Stochastic tree analysis has generated a welcomed union between present value and probability risk estimates. If all of the assumptions inherent in this method are met, it may become the best available operational technique.

5.0 Perhaps, the most important factor affecting the choice of an investment is the cost of capital.

Most differences of opinion, relative to the "costing" of capital has centered about the cost of equity funds and composite funding.

5.1 Of the four different criteria proposed for the evaluation of equity fund costs, this writer recommends Profs. Myron J. Gordon's and Eli Shapiro's model:

In this model,

$$k = \frac{D_o}{P_o} + g$$

where

P_o = a share of stock price at time, $t = 0$

$\overline{D_o}$ = expected dividend at time, t = 0

k = rate of profit or cost

g = rate at which the dividend is expected to grow per year

In effect, this criterion has built upon Criterion 2 of Chapter IV; i.e., $k' = \frac{D_o}{P_o}$, and has extended it by giving due consideration to the prospective growth of a share's revenue. The fact that a share's growth prospects influences the cost of equity funds has been overlooked in the formulation of Criteria 1,2,3,4 of Chapter IV.

5.2 In addition to the above considerations, the E/P ratio criteria, (i.e., the cost of equity capital equals the ratio of current earnings per share to the current market price per share), fails to recognize the fact that the earnings per share are not the payments made to the stock holders.

5.3 An analysis of Prof. Ezra Solomon's criterion of E'/P (i.e., the cost ratio where E' measures the best estimate of the average of the future expected earnings per share if the proposed capital expenditures are not made, and P represents the current market price per share), reveals the following shortcomings:

If the investment is not undertaken, the shares of the company can be expected to sell at a price P_a, corresponding to earnings of E_a and not at a price P. In that case, the relevant capitalization rate is E_a/P_a and not E_a/P. This price P is the amount the firm receives for one new share, and it may differ from the market price of the shares in the near future if the investment does take place.

In effect, it is necessary to estimate both the future earnings, E', and the future market price P' if the investment were to be made, and E_a and P_a if the investment is not made. Then, the decision maker must decide which of the sets (P_a, E_a/P_a) or (P', E'/P') offers the better prospects to the stock holder.

5.4 If an analysis is extended to Prof. Ezra Solomon's proposal for determining the combined cost of debt and equity, several weak points or assumptions are noted:

Making use of Prof. Solomon's notation, we have:

	Market Values	Earning Flows
Stock	S	E
Bonds	B	F
Total Value	B+S=V	0 = E+F

Therefore:

Overall capitalization rate: $k_o = \frac{0}{V}$

Interest on debt: $k_i = F/B$

Equity Capitalization Rate: $k_e = E/S$

Then, we have the identity:

$$k_o = \frac{S}{(B+S)} k_e + \frac{B}{(B+S)} k_i$$

$$= w_1 k_e + w_2 k_i$$

$$= w_1 k_e + (1-w_1) k_i$$

Now, if $\Delta 0$ is the expected benefits from the investment, and C the cost of the investment, the acceptance criterion is $\frac{\Delta 0}{C} > k_o$. However, it should be noted that in order to be in agreement with the previous formulation, this k_o must refer to the cost of capital the company assumes for the future, in the absence of the new investment.

Prof. Solomon argues that by allowing each new proposal a different proportion of debt financing; (i.e., vary w_1 and w_2), it is possible to adjust the overall level of business plus financial uncertainty contained in any given proposal so that the net yield it offers can be compared against the overall uncertainty reflected in k_e. This manipulation of the values of w_1 and w_2 is somewhat contradictory with the fact that the capitalization structure of the firm is supposed to remain unchanged; (i.e., since leverage effects have been omitted).

In addition, a change in the values of w_1 and

w_2 presupposes that the decision maker has a definite, probably correct, idea about the cost of new borrowing.

The restrictive character of the assumptions incorporated in this model, concerning taxes and equality of returns, can also be cited as a point in its disfavor.

5.5 Closely related to the cost of composite financing is the effect of a firm's capital structure.

The two opposing points of view on this subject are:

(a) The traditional approach

(b) The Modigliani - Miller Thesis

A review of both methods follows:

(a) The traditional approach.

This technique, endorsed by this author, can be stated as follows: Other things being equal, the market value of a firm's securities will rise as the amount of leverage in its financial structure is increased from zero to some point determined by the capital market's evaluation of the level of business uncertainty involved. Beyond this point, changes in leverage have very little effect. Outside of this range of "acceptable" leverage the total market value of securities will decline with further increases in leverage. In other words, there is some specific degree of financial leverage at which the market value of the firm's securities will be higher than at other degree of leverage. It is thus assumed that every firm has an optimal point of leverage. But, outside of the theoretical aspects of this concept, it is difficult to isolate this optimum. The reasons for this probably lie in the fact that examinations of leverage effects involve other elements in the financial structure; e.g., kinds of financial structure used by various industries, age of company, managerial reputation, conditions in the capital market, etc.

A corollary of this concept is that a moderate use of debt lowers the total cost of capital. The reason for this is the fact that debt is usually cheaper than equity capital. It is easy to illustrate this by comparing a firm's interest yield on bonds to the earning yield on stock. Stock yields are usually several times larger than bond yields. As the firm's debt ratio increases, however, it would have to offer higher interest rates to compensate bond holders for their increased risk.

(b) The Modigliani - Miller Thesis (The M-M Thesis).

This proposal has as its basic proposition the following:

In a world of perfect markets and rational investors, two identical companies; i.e., two sets of assets offering net operating earnings of the same size and quality, must have the same total market value, regardless of differences in leverage. Thus, it is assumed that the market value of a firm, and hence its cost of capital are both independent of its financial structure.

One can not find fault with the mathematics employed by Profs. Modigliani and Miller. The fault appears to lie with their assumptions:

(1) rational investors

(2) perfect markets

(3) possibility of separating firms into equivalent return classes

(4) personal leverage which does not differ from corporate leverage

The pitfalls in the first three assumptions are rather obvious and do not require any amplification. A brief examination of the forth assumption reveals the following:

The M and M Thesis attempts to demonstrate that the ability of investors to engage in personal leverage is enough to ensure that corporate leverage in itself cannot alter total market value, except for the tax effect factor. In other words, it assumes that personal and corporate leverage can be regarded as equivalent; because for certain conditions, a stock holder in a levered firm will presumably transfer the company's leverage to himself by selling the stock, taking out a personal loan, and investing the total proceeds in the stock of the unlevered firm. The fact that the stock holder is liable for the full amount of his personal loan when he levers himself, compared to his limited liability when he held stock in a levered corporation is completely ignored.

6.0 Perhaps, because profit maximization, as the sole guide to capital budgeting decision making, has recently been under fire, utility theory is becoming entrenched in this activity of capital budgeting.

An alternative to expected profit maximization, the principle of maximizing expected value is favored by many.

6.1 The utility function of a firm really is the utility function of its decision maker(s).

Although many investigators speak of a firm's utility, there really is no such thing. Utility must be associated with a firm's decision maker(s). It is necessary to express <u>his</u> utility function as a key to a firm's action. For example, if a firm's conservative president retires, and is replaced by a younger and more aggresive executive, a new decision maker's utility function will come into being, which is then indicative of a "riskier" attitude on the part of the firm.

6.2 Much research remains to be done before utility theory can successfully be applied to a firm.

The concept of question and answer to lottery quantities and probabilities is about the only way there is of generating a utility function today. The validity and stability of these functions must be proved. Because utility theory will no doubt play a bigger role in the future investment decision world, such research is a must. It would probably be easier to formalize and verify a utility function for a long established organization with well defined goals and funds, such as a public utility company, rather than a very competitive, growing company. Perhaps, this is where such research should start.

7.0 Game theory has been treated as the proverbial stepchild by both the theorists and the decision makers.

Game theory has not prospered and it is unlikely that it will in the near future because of its restrictive assumptions.

The advantages, however, that a decision maker could derive from the use of game theory should not be overlooked; e.g., indifference probabilities.

8.0 After all is said and done, it is possible to view the overall capital investment process as a basic problem in demand and supply.

The demand a proposal exerts for funds is subject to the proposal's profitability, while supply of funds depends on the cost of capital.

If perfect information is available, the capital budgeting problem of the firm could be easily solved by simply using the procedures described in section 4.5 of this thesis. But, because perfect information is almost never available, the capital investment problem explodes into an enormously complex conundrum.

CHAPTER VIII RECOMMENDATIONS FOR FUTURE RESEARCH PROJECTS

The intent of this chapter is to merely list, without much detail, possible areas of future research.

8.1 Risk and Present Worth

Most decision makers would agree that on an overall basis, the present worth method gives the most valid answers. However, in using this method, risk is ignored or considered equal for all proposals. What is needed is a mathematical model which would incorporate risk into a present worth technique. Such a marriage, which no doubt would involve utility theory, would be most welcome and useful. It must also be on a practical and operational level.

8.2 Post Analysis

Once a proposal or project has been adopted and allowed to grow, how does one evaluate the results of this project. This is especially true of Research and Development projects. Ira Horowitz in (63) states, "It is difficult not to be impressed with our inability to evaluate the results of R-D. . . . Many of the possibly well intended attempts to bridge this gap have been crude and have culminated in a series of hopeful, if unjustified, inferences".

It would be necessary to isolate, identify and put into a mathematical relationship, the performance influencing variables of the project. The evaluation of these variables can then take place on different levels:

a. the broad levels of the economy or industry

b. the narrower level of the firm

c. the sharpest level of the individual project

8.3 Maximization of the Utility of Money

Donald E. Farrar (11) developed a mathematical model for optimum selection of a stock portfolio.[1] His basic procedure is to maximize a utility objective function subject to a set of restrictions. Pertinent to his model, his two assumptions are:

a. An investor's utility of money function is positively sloped and concave downward.

b. An investor's strategy is the maximization of expected utility.

It is possible to develop a capital budget model along lines similar to Farrar's portfolio model.

8.4 Stochastic Investment Problems

More research is needed in the field of dynamic programming as it relates to capital investment. Most investment decisions are not isolated activities but extend in time. The acceptance or rejection of any project can affect other projects now or in the future; and in the same fashion, anticipated future projects can affect decisions today. The work done, to date, in this area has been meager. A good general, operational dynamic model for capital budgeting is badly needed.

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[1] Closely related to the capital budget problem are the stock and bond portfolio selection problems. This latter investigation has been carried on by men like Harry Markowitz, Donald E. Farrar, Eugene F. Fama, William F. Sharpe, and Pao Lun Cheng.

8.5 Diversification and Capital Budgeting

What effect does centralization or diversification have on the capital budgeting problem? Where a large organization has several locations at different geographical sites, and if the managers of these locations are fairly independent, what is the best overall capital budgeting process? Should it be completely centralized, decentralized or should only certain expenditures be centralized?

APPENDIX 1

Utility Theory Axioms

From the Viewpoint of Capital Budgeting

AXIOM I

Alternatives shall be comparable. The firm will prefer one or the other of any two alternative outcomes or the firm will be indifferent between them.

AXIOM II

If a lottery[1] option has as one of its outcomes another lottery option, then this first option can be decomposed into the more basic outcomes through the use of ordinary probability calculus.

AXIOM III

If the firm prefers outcome A to outcome B, and outcome B to outcome C, then there exists a lottery option involving outcome A and C which is indifferent, as far as the firm is concerned, to outcome B.

AXIOM IV

If the firm is indifferent between two lottery options, then they are interchangeable as alternatives in any compound lottery.

AXIOM V

A firm's preference and indifference relations for lottery options are transitive.

1 "A lottery ticket is a chance mechanism which yields the prizes A_1, $A_2, \ldots, A_4$ as outcomes with certain known probabilities. ---Operationally, one can think of a lottery as the following experiment: A circle having unit circumference is subdivided into arcs of length p_1, $p_2, \ldots, p_4$ and a "four" pointer is spun which if it comes to rest in the arc of length p_i means that prize A_i is the outcome." (16, p. 24). It should be noted that one and only one prize will be won in a lottery.

AXIOM VI

If two lottery options involve the same two alternatives, then the option in which the more preferred alternative has the higher probability of occurring is itself preferred by the firm.

APPENDIX 2

Six Rules Necessary For the Employment of the Minimax Strategy

A violation of the minimax strategy occurs:

1. If communication between the two players is allowed, then either one might very well attempt to influence the other by threatening, promising, or deceiving.

2. If probabilistic uncertainty, either in the outcome or in the selection of strategies, is introduced, we need to extend our utility function to include rankings for risky events.

3. If an individual does not have a complete and consistent (i.e., transitive) ordering represented by a utility function, the notion of an optimal solution which maximizes utility is ill-defined.

4. If there is not a clear and definite conflict of interests in terms of utility of the outcomes for each player, it cannot be demonstrated that player one, say, should use his saddle-point strategy, even though player two chooses his minimax strategy.

5. If the game does not have a saddle point and player two, say, uses his minimax (pure) strategy, it is not generally advantageous for player one to use his minimax strategy.

6. Finally, if player one, say, knows that player two is not going to use his minimax strategy, it is usually not advantageous for player one to choose his own minimax strategy.

APPENDIX 3

Game Theory--A Brief Review

The first step to carry out in the game theory attack on a problem is to determine the number of players. If the game has only one player, a game of solitaire, there is no need for game theory. The player theoretically selects that alternative which yields the most. If there are elements of chance involved, the player selects that alternative which would bring the best results on the average. However, a one-person game may have two players by considering nature a player. Nature is not a malevolent adversary. Her interests are not necessarily opposed to those of the player. If nature is in accord with the player, there is no conflict. However, when a player is playing against the future (this would be the case in a capital budgeting application), nature is considered the opposing player. Any knowledge about nature that does not oppose the interests of the player can be accounted for, and strict game theory applied to the rest.

More than two-player games are theoretically solvable via game theory. However, potential coalitions and other changing ties of interest greatly complicate the mathematics involved and therefore will not be included here. From a practical standpoint, this type of game may be reduced to a two-player game by considering one player versus all others, collectively.

Two-player games are representative of the common conflict situation. A player, again, is a distinct (and therefore selfish) set of interests.

A strategy is a plan so comprehensive that it cannot lose to the opposing player's action. In a word it is "complete." This means that every possible eventuality of the opposition has been provided for.

The "payoff" is another important consideration of these games. If there is merely an exchange of assets between the two players during the course of the game, then the game is called a "zero-sum game." This means that one player's winnings are the other player's losses. The game is thus a closed system. If, for some reason, one player's gains do not equal another player's losses, then a third player has been introduced, possibly nature. This latter game is called a "non-zero-sum game."

The game is laid out in a "game matrix." The matrix is a cross-hatched grid with the rows representing one player's strategies and the columns representing the other player's strategies. Each individual square, formed by the intersection of a row and a column, represents the outcome of the respective strategies that form the box.

In other words,

> if row one represents player one's first alternative strategy,
>
> and column three represents player two's third alternative strategy,
>
> then the intersection represents what would happen if player one used his first strategy and player two used his third strategy.

This result is the payoff. A positive payoff number means a gain for the player using rows and a loss for the player using columns. A negative number would, of course, be a loss for the row player.

An example of a game matrix for a 5 x 4 game:

COLUMN PLAYER

		1	2	3	4
	1				
	2				
ROW PLAYER	3				
	4				
	5				

The numbers represent the alternative strategies. They have not been quantified, therefore no payoff figures appear in the boxes.

The first major problem of game theory analysis is to get the data into the above form. The second problem is to deduce an answer. Put more simply, the problem must first be defined and then solved.

Defining the problem entails putting numbers into the payoff boxes. These numbers must represent the payoff values of the alternative strategies. To be analyzed, all the numbers must have a common unit. More basically, the real life data must be quantifiable; it must be measurable with sufficient accuracy. When measured, it must not lose its identity with the real world. The value of the final results can be no better than this basic abstraction.

Accordingly, the problems here are: what to measure, and how to base analysis on these measurements. Game theory directly attacks this second problem. It assumes that, in any game matrix, rational players should behave in some definite way. Thus the object of the informed player is to gain as much as possible from a knowledge of the definite optimum strategy. He, of course, assumes that his opponent skillfully pursues an opposite goal.

In terms of the game matrix, the essence of the conflict to be resolved by game theory is as follows: One player wishes to follow that strategy in which the least number he can win is as great as possible, regardless of the other player's strategy. The other player's comparable desire is to make the greatest number he can lose just as small as possible, again, regardless of what his opponent (in this case the first player) may do. A complete knowledge of all the alternatives is assumed for both players. Within this frame, there is a way to solve every two-player game so that each player is optimizing his minimum advantageous asset change.

In a particular game, if the first player picks one strategy, he may win 3 or 4, depending on his opponents move. If he picks his other strategy, he may win 4 or 5, again depending on his opponent. The opponent or second player may choose one strategy where he will lose 3 or 4 or a second strategy where he will lose 4 or 5.

The game matrix for this problem is:

		SECOND PLAYER 1	SECOND PLAYER 2
FIRST PLAYER	1	3	4
	2	4	5

For the first player, the minimum he could gain in the first strategy would be 3. The minimum he could gain in the second strategy would be 4. Therefore the maximum-minimum is 4 and would be guaranteed in the second strategy:

SECOND PLAYER

		1	2	Row Minimum
FIRST PLAYER	1	3	4	3
	2	4	5	4 (maximum-minimum)

For the second player, the maximum to be lost in the first strategy would be 4 and in the second strategy it would be 5. Thus the minimum-maximum would be 4 and would occur in his first strategy:

SECOND PLAYER

		1	2
FIRST PLAYER	1	3	4
	2	4	5
Column Maximum		4 (minimum-maximum)	5

Obviously, the first player should choose strategy number 2 while the second player chooses his first strategy. In this example the maximum-minimum of the first player equals the minimum-maximum of the second player. This is called a "saddle point." It is not uncommon and is an immediate solution (and the correct one).

Also in this game, when played correctly, the first player always gains 4 and the second player always loses 4. This apparent unfairness is quickly remedied by requiring the first player to always pay the second player 4 before each game. This value is known as the value of the game.

Here each player is required to use a single or pure strategy each and every time. This is contrasted with the more frequently occurring mixed strategy which will be considered next.

A mixed strategy game might appear in the following game matrix form:

SECOND PLAYER

		1	2	Row Minimum
FIRST PLAYER	1	4	8	4
	2	7	5	5 (maximum-minimum)
Column Maximum		7 (minimum-maximum)	8	

The largest value for a row minimum is 5 which corresponds to the first player's second strategy. If he goes with this, he knows he will never receive less than 5. Via a similar argument the second player could make sure that he never loses more than 7. Since these two figures are not equal, there is no saddle point. Here there is an unexplored area that upon further application of game theory can reap even more for the skilled player.

If the first player were to stick with the second strategy, then the second player would soon catch on, and the first player would receive only 5. But if the second player sticks with his second strategy, then the first player could change to his first strategy and gain 8.

This develops into quite a problem. Neither player can afford to sit with one strategy because the other will profit unduly. Thus the players must sometimes use one strategy and sometimes the other. Thus to have a complete course of action a player must use a mixed strategy. In this case, if either player knows the other's move in advance, he stands to gain. So the decision as to which choice to make must remain a secret. Thus the choice of which pure strategy to use must be left up to some chance device.

Now the value of this game can be calculated. Odds are determined by some chance device such as flipping a coin. The payoffs for a given strategy of one of the players is weighted by the odds. The payoffs, multiplied by the appropriate odds, are added together and divided by the sum of the odds to determine the expected average value of the game over the long run.

For the first player's first strategy $\frac{(1)(4) + (1)(8)}{1 + 1} = 6$

For his second strategy $\frac{(1)(5) + (1)(7)}{1 + 1} = 6$

Thus the value of the game is 6. In other words, by following a pure strategy the first player would only have been sure of 5. But by following a mixed strategy governed by chance, he can be sure of averaging 6.

Now to move on to a slightly more complicated class of games, the 2 X n games . In these games one player has two strategies while the other has more than two alternatives. The n stands for any integer greater than two.

These 2 X n games may exhibit one significant new concept. The player who has more than two strategies may have some strategies that are self-evidently inferior and therefore should be immediately eliminated. When one of this player's strategies is superior on a box-by-box basis to another, it dominates this latter strategy and therefore the latter strategy may be eliminated from the game matrix.

For example a 2 X n game might appear as follows:

FIRST PLAYER (MAXIMIZING PLAYER)

SECOND PLAYER (MINIMIZING PLAYER)		1	2	3	4	5
	1	3	6	5	7	5
	2	4	3	4	2	5

Comparing the first player's stragegy with his fifth strategy, it is obvious that the fifth strategy is better, box-by-box. Therefore, the first player's first strategy should be eliminated from the matrix.

When this player's third strategy is compared to the fifth, the fifth dominates. The fifth strategy is better than, or at least as good as, strategy number three. Thus this third strategy may be eliminated.

By using this dominance idea, the game is simplified to the following matrix of the 2 X 3 variety.

FIRST PLAYER

		2	4	5
SECOND PLAYER	1	6	7	5
	2	3	2	5

In this example, the second player's first strategy dominates his second strategy. The second player is attempting to minimize his losses, thus it is to his advantage to eliminate the dominating strategy, the first one. The problem then further simplifies to this:

FIRST PLAYER

		2	4	5
SECOND PLAYER	2	3	2	5

But now the problem is all solved, because a saddle-point exists and the value of the game is 5. The first player should always choose his fifth strategy and the second player his second.

In summary, a 2 X 2 game is searched for within the 2 X n game whose solution is also a solution of the more complex game.

More complex games do not require any new conceptual ideas. However, they do require more work to sift and analyze while using no more than extensions of the ideas already explained. For example, let us analyze the following 3 X 3 game:

SECOND PLAYER

		1	2	3	Row Minimum
FIRST PLAYER	1	7	1	5	1
	2	9	11	3	3 (maximum-minimum)
	3	5	7	-1	-1
		9	11	5 (minimum-maximum)	

By inspection it is quickly determined that there is no saddle-point and that the value of the game lies somewhere between 3 and 5. Further inspection displays some dominant pure strategies. For the first player the third strategy is dominated by the second strategy and therefore can be eliminated from the matrix. The second player's third strategy is dominated by his first strategy. In the interests of minimizing his losses the first strategy should be eliminated. Therefore the game simplifies to the following:

SECOND PLAYER

		2	3	First Player's Odds
FIRST PLAYER	1	1	5	8
	2	11	3	4
	Second Player's Odds	2	10	

The odds, the inverse of the column and row differences, can quickly be calculated. These odds are proportional to 1:5 and 2:1 This solution of the simple 2 X 2 game now is expanded the solution of the original 3 X 3 game emerges.

		SECOND PLAYER			
		1	2	3	First Player's Odds
FIRST PLAYER	1	7	1	5	2
	2	9	11	3	1
	3	5	7	-1	0
Second Player's Odds		0	1	5	

This means the first player should mix his strategies according to the odds 2:1:0. 0, of course, means that he shouldn't use this strategy at all. If he is only going to play once, he must use some device that will give these odds, and thus by chance determine the actual choice. Otherwise, his opponent will be ready for him. A similar argument explains the grand or mixed strategy with odds of 0:1:5 to be used by the second player.

The value of the game is determined as explained previously. Here it is calculated for the first player against the second player's second strategy.

$$\frac{(2)(1) + (1)(11) + (0)(7)}{2 + 1 + 0} = \frac{13}{3} = 4\ 1/3$$

Game theory is much more advanced than the presentation given up to this point. However, the author will not pursue the subject any further, since it will not offer any greater assistance with the understanding of Chapter VI.

Excellent references on game theory are (16), (34), (36), (19), (81).

APPENDIX 4

Leading Investigators in Capital Budgeting

Since 1950, capital budgeting has been critically analyzed, evaluated, and developed on conceptual and operational levels. The theorists who have pushed ahead on the conceptual level have been pioneers like Joel Dean, Ezra Solomon, Myron Gordon, Eli Shapiro, Franco Modigliani, Merton H. Miller, James H. Lorie, Leonard J. Savage, Friedrich and Vera Lutz[1], David Durand, and J. Hirshleifer.

On the operational level, the following table lists some of the recently developed techniques:

TABLE VIII

Recently Developed Operational Techniques
For Use In Capital Budgeting

INVESTIGATOR(S)	TECHNIQUE DEVELOPED	REFERENCE
1. A. Charnes, W. W. Cooper, M. H. Miller	Application of Linear Programming To Financial Budgeting and the Costing of Funds.	(45)
2. Rudolph J. Freund	Linear Programming Model Which Incorporates Risk.	(53)
3. H. Martin Weingartner	Integer Programming for the Analysis of Capital Budgeting Problems	(35)
4. George W. Summers	A Mathematical Model for Financing New Firms	(32)

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1 Harry V. Roberts in (75, p. 198) wrote: "Two books of great potential significance for business practice were published in 1951: Capital Budgeting by Joel Dean and Theory of Investment of the Firm by Friedrich and Vera Lutz."

Prior to 1950, two great theorists were J. M. Keynes and Irving Fisher.

INVESTIGATOR(S)	TECHNIQUE DEVELOPED	REFERENCE
5. Frederich S. Hillier	Mathematical Model to Incorporate Risk in Investment by Use of an Estimate of Inexactitude of the Expected Value of a Prospective Cash Flow	(61)
6. R. F. Hespos P. A. Strassman	Stochastic Decision Trees for the Analysis of Investment Decisions	(58)
7. John F. Magee	Use of Standard Decision Trees in Capital Investments	(68)(67)
8. David B. Hertz	The Evaluation of Risk Analysis in Capital Investments	(57)
9. Joel Cord	Dynamic Programming Model for Optimally Selecting Capital Investment With Uncertain Return, and Limited Funds[1]	(46)
10. Sidney W. Hess Harry A. Quigley	Use of Monte Carol Techniques in The Analysis of Risk In Capital Investment	(59)

[1] The concept involved in the analysis are related to Markowitz's work on the portfolio problem; i.e., constraint on the maximum average variance allowed in the final investment package. (Harry M. Markowitz, Portfolio Selection, John Wiley and Sons, 1959).

BIBLIOGRAPHY

Books

1. Ackoff, Russell., Shiv K. Gupta, and Sayer J. Minos, Scientific Method, New York: John Wiley and Sons, Inc., 1962.

2. Barges, Alexander, The Effect of Capital Structure on the Cost of Capital: A Test and Evaluation of the Modigliani and Miller Propositions, (Ford Foundation Doctoral Dissertation Series), Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963.

3. Barish, Norman N., Economic Analysis for Engineering and Managerial Decision Making, New York: McGraw Hill Book Co., Inc., 1962.

4. Bierman, Harold, Jr., and Seymour Smidt, The Capital Budgeting Decision, New York: Macmillan Co., 1960.

5. Bierman, Harold, Jr., Lawrence E. Fouraker, Robert K. Jaedicke, Quantitative Analysis for Business Decisions, Homewood, Illinois: Richard D. Irwin, Inc., 1961.

6. Churchman, C. West, Prediction and Optimal Decision, Englewood Cliffs, N. J.: Prentice Hall, Inc., 1961.

7. Clark, J. M., Competition As A Dynamic Process, Washington, D. C.: Brooking Institute, 1961.

8. Clemens, E. W., Economics and Public Utilities, New York: Appleton, Century-Crofts, Inc., 1950.

9. Dean, Joel, Capital Budgeting, New York: Columbia University Press, 1951.

10. Drucker, Peter F., Management For Results, New York: Harper and Row, Inc., 1964.

11. Farrar, Donald D., The Investment Decision Under Uncertainty, (Ford Foundation Dissertation Series), Englewood Cliffs, N. J.: Prentice Hall, Inc., 1962.

12. Friedman, Milton, Price Theory, Chicago: Aldine Publishing Co., 1962, (A Provisional Text).

13. Grant, Eugene L., and W. Grant Ireson, Principles of Engineering Economy, New York: The Ronald Press Co., 1960.

14. Istvan, Donald F., Capital Expenditure Decisions--How They are Made In Large Corporations, Indiana Business Report No. 33, Indiana: Indiana University Press, 1961.

15. Keynes, John M., The General Theory of Employment and Money, New York: Harcourt, Brace and World, Inc., 1935.

16. Luce, R. Duncan and Howard Raiffa, Games and Decisions, New York: John Wiley and Sons, Inc., 1957.

17. Markowitz, Harry M., Portfolio Selection, New York: John Wiley and Sons, Inc., 1959.

18. Massé, Pierre, Optimal Investment Decisions, Rules For Action And Criteria For Choice, Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1962.

19. McKinsey, J. C. C., Introduction To The Theory Of Games, New York: McGraw-Hill Book Co., Inc., 1952.

20. Merrett, A. J. and Allen Sykes, The Finance And Analysis of Capital Projects, New York: John Wiley and Sons, Inc., 1962.

21. Morris, William T., Management Science In Action, Homewood, Ill.: Richard D. Irwim, Inc., 1963.

22. Pratt, John W., Howard Raiffa, and Robert Schlaifer, Introduction To Statistical Decision Theory, New York: McGraw-Hill Book Co., Inc., 1965.

23. Rautenstrauch and Villers, Budgetary Control, New York: Funk and Wagnalls Co., 1950.

24. Restle, Frank, Psychology of Judgment and Choice, New York: John Wiley and Sons, Inc., 1961.

25. Savage, Leonard J., The Foundations of Statistics, New York: John Wiley and Sons, Inc., 1954.

26. Schlaifer, Robert, Probability and Statistics for Business Decisions, New York: McGraw-Hill Book Co., Inc., 1959.

27. Schwartz, Eli, Corporation Finance, New York: St. Martin's Press, 1962.

28. Shackle, G. L. S., Expectation in Economics, Cambridge, Mass: Cambridge University Press, 1949.

29. Sinclair, Prior, Budgeting, New York: The Ronald Press Co., 1934.

30. Solomon, Ezra, The Theory of Financial Management, New York: Columbia University Press, 1963.

31. Solomon, Ezra, (ed.), The Management of Corporate Capital, Chicago: University of Chicago, The Free Press of Glencoe, 1959.

32. Summers, George W., Financing and Initial Operations of New Firms, (Ford Foundation Doctoral Dissertation Series), Englewood Cliffs, N. J.: Prentice Hall, Inc., 1962.

33. Terborgh, George, Business Investment Policy - A MAPI Study and Manual, Baltimore: Lord Baltimore Press, Inc., 1958.

34. Von Neumann, John, and Oskar Morgenstern, Theory of Games and Economic Behavior, Princeton, N. J.: Princeton University Press, 1944.

35. Weingartner, H. Martin, Mathematical Programming and the Analysis of Capital Budgeting Problems, (Ford Foundation Doctoral Dissertation Series), Englewood Cliffs, N. J.: Prentice Hall, Inc., 1963.

36. Williams, J. D., The Complete Strategyst, New York: McGraw-Hill Book Co., Inc., 1954.

37. Williamson, Oliver E., The Economics of Discretionary Behavior: Managerial Objectives in a Theory of the Firm, (Ford Foundation Doctoral Dissertation Series), Englewood Cliffs, N. J.: Prentice Hall, Inc., 1964.

38. Wixon, Rufus, Modern Business, Alexander Hamilton Institute, New York, 1953.

Articles

39. Adelson, R. M., "Criteria for Capital Investment, An Approach Through Decision Theory Operations", Research Quarterly, Vol. 16, No. 1, 1965.

40. Alchian, Armen A., "The Rate of Interest, Fisher's Rate of Return Over Costs and Keynes' Internal Rate of Return", pp. 67-71 of collection (31).

41. Angell, James W., "Uncertainty, Likelihood and Investment Decisions", The Quarterly Journal of Economics, February, 1960.

42. Bellman, Richard, "Some Applications of the Theory of Dynamic Programming", Journal of the Operations Research Society of America, Vol. 2, August 1954.

43. Bennion, Edward G., "Capital Budgeting and Game Theory", Harvard Business Review, Vol. 34, No. 6, Nov.-Dec., 1956.

44. Bernhard, Richard H., "Discount Methods for Expenditure Evaluation", Journal of Industrial Engineering, Jan.-Feb. 1962.

45. Charnes, A., W. W. Cooper and M. H. Miller, "Application of Linear Programming To Financial Budgeting and the Cost of Funds", pp. 229-255, of collection (31).

46. Cord, Joel, "A Method For Allocating Funds to Investment Projects When Returns Are Subject To Uncertainty", Management Science, Vol. 10, No. 2, January 1964.

47. Dean, Joel, "Measuring the Productivity of Capital", Harvard Business Review, Jan.-Feb. 1954.

48. Dean, Joel and Winfield Smith, "Has MAPI a Place in a Comprehensive System of Capital Control", Journal of Business, Vol. XXVIII, No. 4, October 1965.

49. Drucker, Peter F., "Long Range Planning", Management Science, April 1959.

50. Durand, David, "The Cost of Capital In An Imperfect Market: A Reply to Modigliani and Miller", pp. 182-197, of collection (31).

51. Eisner, Robert, "Depreciation Under the New Tax Law", Harvard Business Review, Jan.-Feb. 1955.

52. Findler, Nicholas V., "Human Decision Making Under Uncertainty and Risk: Computer Based Experiments and a Heuristic Simulation Program Proceedings", Fall-Joint Computer Conference, 1965.

53. Freund, Rudolph J., "The Introduction of Risk Into a Programming Model, Econometrica, July 1956.

54. Friedman, Lawrence, "Decision Making in Competitive Situations", Management Technology, Vol. 1, No. 2, Dec. 1960.

55. Gordon, Myron J., "The Payoff Period and the Rate of Profit", pp. 48-55 of the collection (31).

56. Gordon, Myron J. and Eli Shapiro, "Capital Equipment Analysis: The Required Rate of Profit", pp. 141-149, of collection (31).

57. Hertz, David B., "Risk Analysis of Capital Investment", Harvard Business Review, Jan.-Feb. 1964.

58. Hespos, R. F. and R. A. Strassman, "Stochastic Decision Trees for the Analysis of Investment Decisions", Management Science, Vol. 2, No. 10, August 1965.

59. Hess, Sidney W. and Harry A. Quigley, "Analysis of Risk in Investment Using Monte Carlo Techniques", Chemical Engineering Progress Symposium Series, Vol. 59, No. 42, 1964.

60. Hill, Horace, Jr., "A New Method of Computing Rate of Return On Capital Expenditures", pp. 35-43 of the collection (31).

61. Hillier, Frederick S., "The Derivation of Probabilistic Information for the Evaluation of Risky Investments", Management Science, April 1963.

62. Hirshleifer, Jack, "On the Theory of Optimal Investment Decision", Journal of Political Economy, Vol. LXVI, No. 5, August 1958.

63. Horowitz, Ira, "Evaluation of the Results of Research and Development: Where We Stand", IEEE Transactions of Engineering Management, June 1963.

64. Klein, M., "A Management Pitfall: Decisions By the Rate of Return Method", Chemical Engineering Progress, Vol. 60, No. 9, September 1964.

65. Lefkovits, H. C., H. Kanner and R. B. Harbottle, "On Multiple Rates of Return", Proceedings of the 5th World Petroleum Congress, 1959.

66. Lorie, James H. and Leonard J. Savage, "Three Problems in Rationing Capital", pp. 56-66 of collection (31).

67. Magee, John F., "How To Use Decision Trees in Capital Investment", Harvard Business Review, Sept.-Oct. 1964.

68. Magee, John F., "Decision Trees for Decision Making", Harvard Business Review, July-August 1964.

69. McLean, John G., "How to Evaluate New Capital Investments", Harvard Business Review, Nov.-Dec. 1958.

70. Modigliani, Franco and Merton H. Miller, "The Cost of Capital, Corporation Finance and the Theory of Investment", pp. 150-181 from collection of (31).

71. Mosteller, Frederick, Philip Nogee, "An Experimental Measurement of Utility", The Journal of Political Economy, Vol. LIX, No. 5, Oct. 1951.

72. Ravencroft, Edward A., "Return on Investment: Fit the Method to Your Need", Harvard Business Review, March-April 1960.

73. Renshaw, Ed, "A Note on the Arithmetic of Capital Budgeting Decisions", pp. 80-88 of collection (31).

74. Reul, Ray I., "Profitability Index for Investments", Harvard Business Review, July-August 1957.

75. Roberts, Harry V., "Current Problems in the Economics of Capital Budgeting", pp. 198-201, of collection (31).

76. Robinson, Romney, "The Rate of Interest, Fisher's Rate of Return Over Costs of Keynes' Internal Rate of Return: Comment", pp. 72-73 of collection (31).

77. Solomon, Ezra, "Measuring a Company's Cost of Capital", pp. 18-30 of collection (31).

78. Solomon, Ezra, "The Arithmetic of Capital Budgeting Decisions", pp. 74-79 of collection (31).

79. Soper, C. S., "The Marginal Efficiency of Capital: A Further Note", The Economic Journal, Vol. LXIX, No. 273, March 1959.

80. Terborgh, George, "Some Comments on the Dean-Smith Article on the MAPI Formula", pp. 307-309 of collection (31).

81. Wagner, Harvey M., "Advances in Game Theory", American Economic Review, June 1958.

82. Fisher, James L., "A Class of Stochastic Investment Problems", Unpublished Doctor of Philosophy Thesis, Case Institute of Technology, 1960.

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